Is There A Shortfall in Public Sector Capital? An Asset Pricing Appraisal

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Abstract

I assess the overall supply of public sector capital in the U.S. through the lens of asset prices. Using a two-sector general equilibrium model, I demonstrate how the supply of public sector capital may become a source of priced risk, for which the price of risk changes sign as public sector capital becomes over- or under-supplied. Taking two complementary empirical approaches, I find consistent results suggesting that assets with higher sensitivity to variations in public investment have higher average returns. Together my findings imply that public sector capital is *undersupplied*, and greater public investment is favorable for investors.

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Public sector capital is an essential underpinning of the economy; its maintenance and enhancement require a significant amount of public investment. [1] [2] In recent years, as stories of crumbling infrastructure abound, there seems to be a growing notion that public sector capital is undersupplied and greater public investment is needed. However, existing studies provide little evidence of a shortfall in public sector capital, and there is a lot of controversy on the potential impact of increasing public investment.

In this paper, I take a novel approach to this question: I infer from asset prices investors' opinion on the overall supply of public sector capital. The basic idea is as follows. If public sector capital is undersupplied, then investors may view the declines in public investment as a source of risk; hence, ceteris paribus, assets that covary positively (negatively) with public investment would be valued lower (higher) and have higher (lower) average returns. I formalize this idea using a two-sector general equilibrium (GE) model in which public sector capital (as a share of aggregate capital) enters the pricing kernel; its price of risk turns positive (negative) when it becomes too low (high). Prompted by this GE theory, I propose a factor pricing model with shocks to the public sector investment share (henceforth, "PUB shocks") as a risk factor. I confront the factor model with a variety of test assets and find that exposure to PUB shocks is priced and carries a robustly *positive* price of risk; this finding suggests an *undersupply* of public sector capital. In addition, I find supporting evidence from the analysis of a sample of U.S. government contractors. Specifically, I find that firms with heavier reliance on government as a customer are more sensitive to changes in public investment and provide higher stock returns on average. I also find that the spread in average returns between firms with high and low government dependency has widened as the public sector investment share declines. Together these findings are consistent with the view that public sector capital is in short supply, and greater public investment is favorable.

For starters, I briefly review the evolution of public sector investment in the United States, comparing it with that of private sector (nonresidential) investment. On average, national investment (private plus public sector investments) represents about 12% of gross domestic product (GDP) in the postwar U.S. economy, of which roughly one third is public sector investment. The latter ratio, which I refer to as the *public sector investment share*, has witnessed significant variations: as shown in Figure 1, it increased in the

[1] In this paper, I use the terms "public investment" and "public sector investment" interchangeably, both of which refer to government spending on public sector (nondefense) capital such as highways, roads, airports, mass transit systems, water and sewer systems, electric and gas facilities, public schools and hospitals facilities; the precise empirical definition is provided later. In the literature, such spending is also referred to as "infrastructure spending or investment", "public capital or fixed investment", and so on.

[2] Munnell (1990) surveys some early studies on the importance of public sector capital as well as the (in)adequacy of public investment.

1950s, peaked in the early 1960s, and has since been trending downward. The most recent reading shows a new record low of less than 15%, meaning that the size of public sector investment is merely one sixth of that of private sector investment. [3]

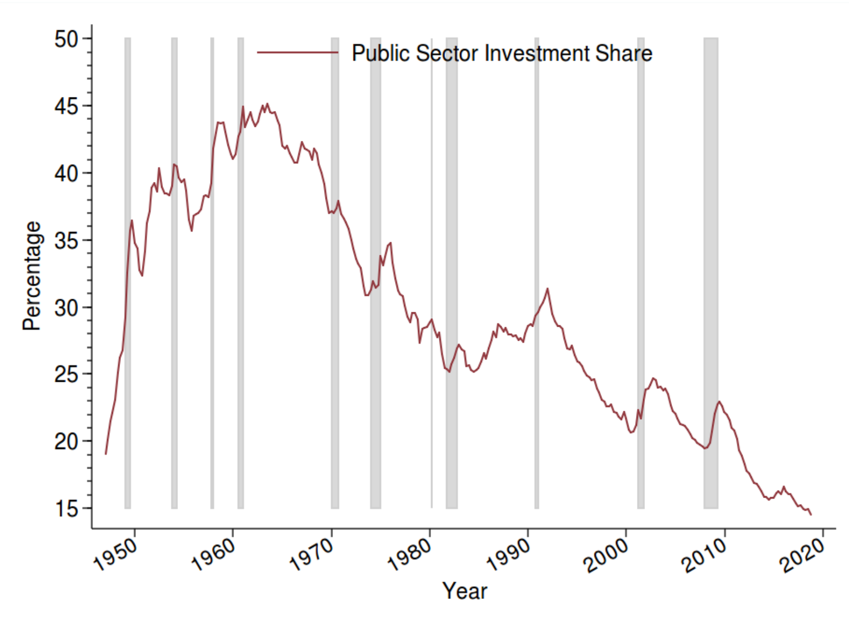


Figure 1: **Public sector investment share.** The solid line represents the public sector investment share, that is, the ratio of public sector (nondefense) investment to the sum of public and private sector (nonresidential) investments; Shaded areas indicate U.S. recessions identified by NBER. Related variables are more precisely defined in Section 3.

Looking at these variations, one might naturally ask whether they have any bearing on the economy, and in particular, whether the level of public investment is appropriate or not. *A priori*, it is hard to answer these questions because, although public investment provides many benefits (Munnell, 1990), it incurs nontrivial costs as well—whether it is the crowding-out of private sector investment (Aschauer, 1989a) or a heavier fiscal burden (Baxter and King, 1993). The fact that public investment has declined relative to the rest of the economy does not in itself indicate that it is inadequate. Hence more evidence is required to make a judgement. Existing studies take various approaches to this problem, but there is little consensus among them. [4]

So I propose a distinctive approach by letting investors speak to this matter. In standard asset pricing theory, investors dislike risks that reduce their utility and value claims

[3]Alternatively, one can use GDP as the denominator when defining the public sector investment share, the behavior of which turns out to be very similar (see Figure B.3).

[4]For example, Haughwout (2002) estimates the marginal benefit of public capital from local wages and housing prices and find it to be small relative to the cost. However, Albouy and Farahani (2017) reinterpret Haughwout (2002)'s estimates through the lens of a more general model and find public capital to be much more valuable.

that hedge them. I hypothesize that investors care about public investment and would like to hedge against its declines (increases) if public sector capital is undersupplied (oversupplied); that translates to higher (lower) risk premia for assets that covary with public investment. I begin by providing theoretical support for this hypothesis.

To theoretically link public investment to investors' utility and thus to asset prices, I develop a parsimonious GE model. I consider a two-sector production economy with the following ingredients. First, I postulate a neoclassical aggregate production function with constant elasticity of substitution (CES); it takes private and public sector capital as inputs. Second, I incorporate time-varying uncertainty as a driver of business cycles and posit a risk-mitigating role for the public sector. As a result of these two features, expanding public sector capital has influence on the aggregate output as well as its variability. Finally, living in this economy is a representative agent who I assume has recursive preferences. Her utility is directly driven by economic prospects, which in turn are determined by aggregate productivity and volatility. So if without any friction, the agent would always hold the supply of public sector capital to an optimal level at which the best economic prospects are achieved.

However, as I introduce two types of frictions, the agent can no longer maintain this optimum. One friction is capital adjustment costs, which prevent instantaneous capital reallocation. Another friction is a constant public investment rule, which renders the public sector investment rate irresponsitve to changing economic conditions. [5] Due to these frictions, public sector capital can deviate from its optimal level, becoming over- or under-supplied.

In this setting, I examine the asset pricing role of a crowding-out shock that increases public sector capital accumulation but leads to an equivalent reduction in private sector capital accumulation. [6] When public sector capital is oversupplied, this shock pushes the capital allocation *away* from the optimum and thus decreases the agent's utility. This results in a negative price of risk for crowding-out shocks. When public sector capital is undersupplied, however, a crowding-out shock pushes the capital allocation *toward* the optimum and thus increases the agent's utility. This leads to a positive price of risk for crowding-out shocks. Therefore, a key implication from this GE model is that the over- or under-supply of public sector capital is associated with different signs for the price of risk

[5]This public investment rule is motivated by the fact that, though the public sector investment share has varied considerably, the growth rate of public sector investment is fairly stable over time (see Figure B.4). Gali (1994) considers a similar rule.

[6]This shock is motivated by Aschauer (1989a)'s finding that an increase in public capital accumulation induces an almost dollar-for-dollar reduction in private capital accumulation. Cohen, Coval, and Malloy (2011) provide another study documenting the crowding-out effect of government spending.

for crowding-out shocks. This insight underpins my empirical investigation in which I try to identify the sign for the price of crowding-out risk.

Admittedly, there are other mechanisms as to how public investment may affect the economy and thus investors' utility. But I focus on the productivity effect and the risk-mitigating effect for good reason. I consider the productivity effect for its predominance in the literature as well as its practical relevance. As pointed out by Blanchard (2016), "U.S. government borrowing costs are very low... the relevant opportunity cost of public investment would not be the rate on government bonds but the marginal product of the private capital that would be crowded out." I incorporate the risk-mitigating effect to match the countercyclicality of the public sector investment share, a salient pattern shown in Figure 1. Underlying this pattern is the fact that private sector investment is much more procyclical than public sector investment. It is important to have the risk-mitigating effect to endogenously generate enough procyclicality for private sector investment.

The equilibrium pricing kernel in this GE model is driven by shocks to the share of public sector capital, economic uncertainty, and the aggregate capital growth. Prompted by this pricing kernel, I propose a three-factor asset pricing model with PUB shocks, uncertainty shocks, and the market excess return as risk factors. PUB shocks—which is a proxy for crowding-out shocks—may stem from, for example, unforeseen fiscal developments. Uncertainty shocks represent news that alter the variability of economic conditions. The market excess return captures standard technology shocks that affect general economic growth. [7] This factor model underpins my empirical investigation. [8]

Guided by the GE model, I go on to investigate whether, in practice, investors really care about the supply of public sector capital to the extent that they might demand hedges against unfavorable changes in public investment, and if yes, what changes are considered unfavorable, increase or decrease? To answer these questions, I empirically estimate the price of risk for PUB shocks. Equipped with the factor pricing model derived from the GE theory, I perform standard two-pass asset pricing tests using a variety of well-known equity portfolios. My main finding is that assets' exposure to PUB shocks possess significant explanatory power for cross-sectional differences in average asset returns, and that the estimated price of risk for PUB shocks is *positive*. This finding points to *increases* in public investment as good news.

To strengthen and extend this finding, I propose a characteristic-based measure to

[7]He, Kelly, and Manela (2017) also use the market excess return as a proxy for the Total-Factor-Productivity-style persistent technology shocks.

[8]It is worth emphasizing that this factor model is actually more general than the GE framework presented here. One may come up with alternative frameworks in which the equilibrium pricing kernels are determined by the same set of state variables.

capture firms' sensitivity to PUB shocks, and I form portfolios based on that. [9] I examine a sample of U.S. government contractors. I postulate that the extent to which a firm depends on government for revenue is a relevant proxy for its covariation with public investment. I form stock portfolios based on firms' government dependency, which is measured by the average fraction of sales to government over the past three years. I find that high-dependency firms are more sensitive to changes in public investment and provide higher stock returns on average compared with low-dependency firms. A zero-investment portfolio that is long stocks in the highest dependency quintile and short stocks in the lowest dependency quintile provides an average return of 7.4% annually. I confirm that this return spread is not driven by differential loadings on classic risk factors. Lastly, I conduct a subsample analysis and find that the spread in average returns between high- and low-dependency firms was small, or even negative, in the 1980s and 1990s, but it has widened considerably in recent years and looks to continue. Together these findings support the view that there is a shortfall in public sector capital, and greater public investment is favorable; this appears particularly true in recent years.

Related literature. This paper contributes to a substantial literature studying the economic effects of public investment. Since the seminal work by Aschauer (1989a,b), a lot of research has been dedicated to understanding the mechanisms by which public investment influences the economy and, in particular, whether the overall impact is positive or negative. Some studies examine public investment at the aggregate level, while others focus on specific types of investments. [10] In any case, the common goal of these studies is to estimate the value of public sector capital, which together with the information on its potential costs help answer the normative question of whether government should increase or decrease public sector investment. Compared with existing studies, I take a novel approach to this question, inferring investors' opinion on this matter from asset prices. [11] I demonstrate that shocks to the public sector investment share are a source of risk that is priced in the cross section of expected returns and carries a positive price of risk. It suggests that investors' utility declines when public investment dwindles; assets

[9] It is well known that characteristics often give a better proxy for firms' risk exposure (Adrian, Etula, and Muir, 2014).

[10] For example, Haughwout (2002) and Albouy and Farahani (2017) study the value of public goods and infrastructure in particular. Cellini, Ferreira, and Rothstein (2010) study public school facilities investment. Allen and Arkolakis (2019) examine transportation infrastructure. McGraw (2018) focuses specifically on airline hubs.

[11] This approach has been used to study various issues, including globalization (Barrot, Loualiche, and Sauvagnat, 2019), inequality (Johnson, 2012), market-wide liquidity (Pastor and Stambaugh, 2003), financial intermediary leverage (Adrian, Etula, and Muir, 2014; He, Kelly, and Manela, 2017), macro uncertainty and volatility (Dew-Becker, Giglio, and Kelly, 2019), and technological growth (Garleanu et al., 2012).

that pay off in this case are considered valuable hedges and hence deliver lower average returns.

My work also relates to Belo and Yu (2013), who made the first attempt to link public investment to the stock market. I extend their work and demonstrate how public sector capital may enter the pricing kernel in general equilibrium. The model in this paper stems from a strand of macro-finance literature that studies the joint dynamics of macro quantities and asset prices in a GE framework. Pioneering work by Jermann (1998) and Tallarini (2000) examines time-inseparable preferences (habit formation preferences and recursive preferences, respectively) in this framework and has achieved some success in reconciling business-cycle regularities with asset pricing facts. Their models are extended in various ways to address many issues, among which Eberly and Wang (2011)'s two-sector model is the most similar to mine. Our main difference is that, in their model, capital from the two sectors are perfect substitutes, whereas in my model, they bear a certain degree of complementarity.

Outline. The remainder of this paper is structured as follows. Section 1 introduces a two-sector general equilibrium model that demonstrates the asset pricing role of PUB shocks. Section 2 discusses the main implications of the model. To investigate how public investment is reflected in asset prices, Section 3 takes to data a factor pricing model derived from this GE theory, and Section 4 conducts a portfolio analysis using a sample of U.S. government contractors. Section 5 concludes. Appendix A and B provide supplementary details and results.

1 Model

In this section, I lay out a two-sector general equilibrium model that establishes the link between the over- or under-supply of public sector capital and asset prices. I also outline the main steps in deriving the solution.

1.1 Setup

I consider a two-sector production economy cast in continuous time with an infinite horizon. An infinitely lived representative agent with recursive preferences presides over this economy, whose objective is to maximize her expected lifetime utility. The private and public sectors—denoted by p and g, respectively—accumulate capital independently. A single type of good is produced via an aggregate production technology with capital from both sectors as inputs. This produced good can be either consumed right away or transformed into capital and installed in either sector. Figure 2 provides a schematic representation of the basic model structure. Details on each element are provided next.

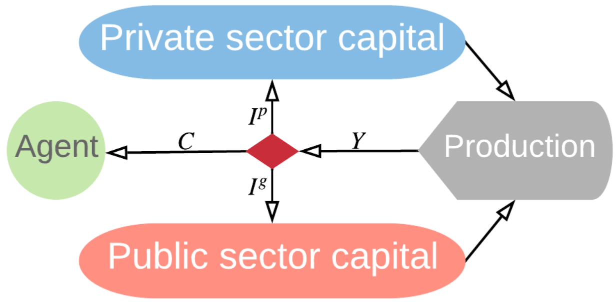


Figure 2: **Schematic model structure.**

Aggregate production. I consider an aggregate production technology that employs private and public sector capital as separate inputs. It produces a final good at a rate of Y_t per unit of time, where $Y_t = F(K_t^p, K_t^g)$ is specified as a constant elasticity of substitution (CES) function

$$m\left[\alpha(K_t^p)^{\frac{s-1}{s}} + (1-\alpha)(K_t^g)^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}, \quad (1)$$

in which K_t^p and K_t^g denote the stocks of private and public sector capital, respectively. The parameter α determines the output-maximizing allocation of capital between the private and public sectors, m the scale, and s the elasticity of substitution. [12]

It is worth mentioning that, to model government's contribution to production, existing studies consider either the current flow of government spending (e.g., Barro, 1990) or the accumulated stock of public sector capital (e.g., Baxter and King, 1993) as an additional input into the production function. Because the government input considered here is intended to represent productive capital such as infrastructure, I adopt the accumulated stock approach.

For the convenience of exposition as well as equilibrium characterization, I conduct a change of variables. I define $K_t \equiv (K_t^p + K_t^g)$ as the aggregate stock of capital, and $\chi_t \equiv \frac{K_t^g}{K_t}$ as the fraction accounted for by public sector capital. Accordingly, the output rate Y_t can

[12] If $s \to 0$, private and public sector capital become perfect complements. If $s \to 1$, this function converges to the popular Cobb-Douglas function. If $s \to \infty$, private and public sector capital become perfect substitutes.

be rewritten as $M(\chi_t)K_t$, where $M(\chi_t)$ is given by

$$M(\chi_t) = m\left[\alpha(1-\chi_t)^{\frac{s-1}{s}} + (1-\alpha)\chi_t^{\frac{s-1}{s}}\right]^{\frac{s}{s-1}}. \tag{2}$$

$M(\chi_t)$ has an interior maximum—that is, $\exists\chi^*$ such that $M(\chi^*) \geq M(\chi)$ for $\forall\chi \in (0,1)$. At the maximum, the marginal products of private and public sector capital, which are given by

$$r_t^p = -\chi_t M'(\chi_t) + M(\chi_t) \qquad\qquad r_t^g = (1-\chi_t)M'(\chi_t) + M(\chi_t), \tag{3}$$

are equalized. [13] Thus, for a given amount of aggregate capital K_t, the maximum output is attained when a certain fraction χ^* of capital is allocated to the public sector; having either too much or too little public sector capital would lead to less output.

Capital accumulation. Private and public sector capital evolve according to

$$\frac{dK_t^p}{K_t^p} = [\phi(\iota_t^p) - \delta]dt + \sigma_{1,t}^p dZ_t - \sigma_{2,t}^p dW_t \qquad \frac{dK_t^g}{K_t^g} = [\phi(\iota_t^g) - \delta]dt + \sigma_{1,t}^g dZ_t + \sigma_{2,t}^g dW_t, \tag{4}$$

where $\iota_t^p \equiv I_t^p/K_t^p$ and $\iota_t^g \equiv I_t^g/K_t^g$ are investment-capital ratios, and δ is the depreciation rate. [14] As is standard in the literature, I assume that capital investment incurs adjustment costs: investing in sector $i \in \{p, g\}$ at a rate of $\iota_t^i K_t^i$ per unit of time can sustain an *expected* capital growth rate of $\phi(\iota_t^i)$ before depreciation. Function $\phi(\cdot)$, which satisfies $\phi'(\cdot) > 0$ and $\phi''(\cdot) < 0$, represents a classic investment technology with adjustment costs. [15] It imposes higher costs on rapid changes to capital.

I consider two mutually independent Wiener processes, Z and W, as sources of exogenous shocks that drive capital accumulation and allocation. Without loss of generality, I assume that: (1) $\sigma_{1,t}^p = \sigma_{1,t}^g = (1-\chi_t)\sigma_t$; (2) $\sigma_{2,t}^p = \chi_t\varsigma$ and $\sigma_{2,t}^g = (1-\chi_t)\varsigma$. As a result, I

[13] Suppose the amount of private sector capital increases by ϵ, and then the aggregate output would become $M(\frac{K_t^g}{K_t+\epsilon})(K_t+\epsilon)$. Taking derivative w.r.t. ϵ and evaluating at $\epsilon = 0$, I obtain

$$\left.\frac{\partial M(\frac{K_t^g}{K_t+\epsilon})(K_t+\epsilon)}{\partial\epsilon}\right|_{\epsilon=0} = -\chi_t M'(\chi_t) + M(\chi_t)$$

which is r_t^p. A similar calculation gives r_t^g. Notice that $M(\chi_t)K_t - r_t^p K_t^p - r_t^g K_t^g = 0$.

[14] I use the same depreciation rate for private and public sector capital because data are generally unavailable to produce a comprehensive measure of government inventory depreciation (U.S. Bureau of Economic Analysis, 2019). Besides, this parameter has little impact on my results.

[15] I adopt from Jermann (1998) the capital adjustment cost function, $\phi(\iota) = \varphi_0 + \frac{\varphi_1}{1-1/\varrho}\iota^{1-1/\varrho}$.

obtain the processes for K_t and χ_t:

$$\begin{aligned}\frac{dK_t}{K_t} &= \underbrace{[(1-\chi_t)\phi(\iota_t^p) + \chi_t\phi(\iota_t^g) - \delta]}_{\mu_{K,t}} dt + \underbrace{(1-\chi_t)\sigma_t}_{\sigma_{K,t}} dZ_t \\ d\chi_t &= \underbrace{\chi_t(1-\chi_t)[\phi(\iota_t^g) - \phi(\iota_t^p)]}_{\mu_{\chi,t}} dt + \underbrace{\chi_t(1-\chi_t)\varsigma}_{\sigma_{\chi,t}} dW_t.\end{aligned} \tag{5}$$

I allow uncertainty σ_t to vary over time according to

$$d\sigma_t = \kappa(\bar{\sigma} - \sigma_t)dt + \nu\sqrt{\sigma_t}dZ_t^{\sigma}, \tag{6}$$

where κ controls the speed of mean-reversion, $\bar{\sigma}$ is the long-run mean, and ν governs the variability of σ_t. I introduce another Wiener process Z^{σ} to generate uncertainty shocks. I assume $dZ_t \cdot dZ_t^{\sigma} = \rho_{K\sigma}dt$ with $\rho_{K\sigma} < 0$ in all cases; this is in accordance with the suggestion of Bloom et al. (2018), who argue that recessions are best modeled as a combination of negative first-moment shocks (dZ_t) and positive second-moment shocks (dZ_t^{σ}).

This setting permits a clear interpretation of the shock processes. Innovations in process Z capture standard technology shocks that affect general economic (capital) growth. Innovations in process Z^{σ} are uncertainty shocks that alter the variability of economic conditions. Innovations in process W represent capital (re)allocation shocks that drive the relative shares of private and public sector capital. In particular, a positive realization of dW increases public sector capital accumulation while leads to an equivalent reduction in private sector capital accumulation; it accords with Aschauer (1989a)'s finding of a complete crowding-out of private by public sector capital. The asset pricing role of W-shocks is my primary interest; I will refer to them as PUB shocks hereafter.

Preferences and resource constraint. The representative agent has recursive preferences with the time discount β, the elasticity of intertemporal substitution (EIS) ψ, and the relative risk aversion (RRA) γ:

$$V_t = \mathbb{E}_t \int_t^{\infty} u(C_\tau, V_\tau)d\tau \quad \text{with} \quad u(C,V) \equiv \frac{\beta(1-\gamma)V}{1-1/\psi}\left\{\frac{C^{1-1/\psi}}{[(1-\gamma)V]^{\frac{1-1/\psi}{1-\gamma}}} - 1\right\}, \tag{7}$$

where $\mathbb{E}_t$ is an expectation operator conditional on time-t information. As is well known, recursive preferences allow a separation between the EIS and the RRA. The agent's objective is to maximize utility while subject to the resource constraint

$$C_t + I_t^p + I_t^g = M(\chi_t)K_t. \tag{8}$$

Discussion. Two critical assumptions of the model merit further discussion. First, I assume that augmenting the stock of public sector capital raises the marginal product of private sector capital, and vice versa. This assumption, which underpins the CES production function (1), stems from a substantial literature on the productivity effect of public investment. In particular, the seminal work by Aschauer (1989a,b) finds that public sector capital has nontrivial influence on aggregate productivity: increasing the stock of public sector capital contributes to the marginal product of private sector capital. His finding has gained traction in the literature, and subsequent studies generally come to the same conclusion (despite some disputes on the magnitude of the effects). [16] [17] Second, I postulate a risk-mitigating role for the public sector. Under this assumption, an expansion in the share of public sector capital (χ_t) reduces the aggregate volatility, $(1-\chi_t)\sigma_t$. This risk-mitigating assumption is motivated by the literature on government size and macroeconomic stability (Gali, 1994; Fatas and Mihov, 2001). In particular, Fatas and Mihov (2001) document a strong negative correlation between government size and macroeconomic variability; the results hold regardless of the measures and are robust both for OECD countries and across states in the U.S.. With these two assumptions, the model captures two important considerations—that is, the influence on productivity and stability—in determining the appropriate supply of public sector capital.

1.2 Solution

I solve the model in two steps. First, I obtain the optimal consumption-investment policy by working out the central planning problem. Then I derive equilibrium conditions that connect macro quantities to prices. Substituting the optimal policy into the equilibrium conditions enables me to express all quantities and prices as functions of the state variables. The following summarizes the key solution steps; omitted details and proofs are given in Appendix.

[16] See, for example, Munnell (1992); Holtz-Eakin (1994); Arslanalp, Bornhorst, Gupta, and Sze (2010).

[17] Anecdotal evidence suggests that the productive role of public sector capital continues to be relevant. For example, Gopalswamy and Rathinam (2018) propose a new approach to autonomous driving that involves upgrading the road infrastructure. They argue that, by taking some responsibility off the shoulders of car manufacturers, this approach can "accelerate the deployment of autonomous driving and correspondingly reap its benefits."

Central planning. In this model, the state of the economy can be summarized by three variables: the aggregate capital stock K_t, the share of public sector capital χ_t, and the level of economic uncertainty σ_t. The first variable merely controls the scale of the economy, while the last two are the effective state variables that determine economic prospects (or, equivalently, investment opportunities). Providing the current state of the economy, the representative agent chooses the consumption-investment policy to maximize her expected lifetime utility

$$V(\chi_t, \sigma_t, K_t) = \max_{\iota_t^p,\, C_t} \mathbb{E}_t \int_t^\infty u(C_\tau, V_\tau) d\tau$$

subject to (5) and (6) as well as (8). The model is homogeneous in scale, so I conjecture that the representative agent's value function takes the form of

$$V(\chi_t, \sigma_t, K_t) = \frac{(\xi_t K_t)^{1-\gamma}}{1-\gamma}, \tag{9}$$

where $\xi_t \equiv \xi(\chi_t, \sigma_t)$ is a function to be determined. I interpret ξ_t as a welfare multiplier that gauges the influence of future economic prospects on the ex ante lifetime utility. Good economic prospects—that is, an optimal allocation of capital and low economic uncertainty—contribute to a large ξ_t, meaning that the agent expects to derive a higher lifetime utility given the current stock of capital. The process followed by ξ_t can be obtained using Ito's lemma:

$$\frac{d\xi_t}{\xi_t} = \mu_{\xi,t} dt + \sigma_{1,t}^{\xi} dZ_t^{\sigma} + \sigma_{2,t}^{\xi} dW_t, \tag{10}$$

where $\{\mu_{\xi,t},\, \sigma_{1,t}^{\xi},\, \sigma_{2,t}^{\xi}\}$ are determined in equilibrium. The HJB equation associated with the central planning problem is given by

$$\begin{aligned}\frac{\beta}{1-1/\psi} = \max_{\iota_t^p,\, \iota_t^g} \frac{\beta}{1-1/\psi}\left(\frac{c_t}{\xi_t}\right)^{1-1/\psi} + \mu_{K,t} + \mu_{\xi,t} - \frac{\gamma}{2}[(1-\chi_t)^2\sigma_t^2 + (\sigma_{1,t}^{\xi})^2 + (\sigma_{2,t}^{\xi})^2] \\ + (1-\gamma)[\rho_{K\sigma}(1-\chi_t)\sigma_t\sigma_{1,t}^{\xi}],\end{aligned} \tag{11}$$

where I define $c_t \equiv [M(\chi_t) - \iota_t^p(1-\chi_t) - \iota_t^g \chi_t]$ as the consumption-capital ratio. The optimal private investment policy is pinned down by the first-order condition,

$$\left(\frac{c_t}{\xi_t}\right)^{1/\psi} = \frac{\beta}{\phi'(\iota_t^p)} \frac{1}{\xi_t - \chi_t\, \partial_\chi \xi_t}.^{18} \tag{12}$$

In the benchmark case, I posit a constant public investment rate, $\iota_t^g = \overline{\iota^g}$, which renders the expected growth of public sector capital irresponsive to changing economic conditions. This is motivated by the fact that the average growth of public sector investment has been fairly stable over time (see Figure B.4). [19] Combining (11) and (12) gives a system of partial differential equations on $\xi(\chi_t, \sigma_t)$ that is solved using an iterative method. The details of this procedure are given in Appendix B. With the solution for $\xi(\chi_t, \sigma_t)$, the optimal private investment policy $\iota^p(\chi_t, \sigma_t)$ can be obtained.

Lastly, the equilibrium pricing kernel is a function of the state variables (its exact expression is in Appendix A)

$$\Lambda_t \equiv \Lambda(\chi_t, \sigma_t, K_t), \tag{13}$$

and its law of motion is given by

$$\frac{d\Lambda_t}{\Lambda_t} = -r_t dt - \eta_t^K dZ_t - \eta_t^\sigma dZ_t^\sigma - \eta_t^\chi dW_t, \tag{14}$$

where r_t is the risk-free interest rate and $\{\eta_t^K, \eta_t^\sigma, \eta_t^\chi\}$ represent the risk prices for process Z, Z, and W, respectively. The expressions for the risk prices are given by

$$\begin{aligned} \eta_t^K =& \gamma(1-\chi_t)\sigma_t \\ \eta_t^\sigma =& \overbrace{\left[(\gamma - 1/\psi)\frac{\partial_\sigma \xi_t}{\xi_t} + 1/\psi \frac{\partial_\sigma c_t}{c_t}\right]}^{-\partial_\sigma \Lambda_t/\Lambda_t} \nu\sqrt{\sigma_t} \\ \eta_t^\chi =& \underbrace{\left[(\gamma - 1/\psi)\frac{\partial_\chi \xi_t}{\xi_t} + 1/\psi \frac{\partial_\chi c_t}{c_t}\right]}_{-\partial_\chi \Lambda_t/\Lambda_t} \sigma_{\chi,t} \end{aligned} \tag{15}$$

Consider an asset that is priced by this equilibrium pricing kernel, its expected excess

[18] The partial derivative $\frac{\partial^n Y}{\partial X_1 X_2 \ldots X_n}$ is denoted by $\partial_{X_1 X_2 \ldots X_n} Y$.

[19] As a comparison, I also solved the model under the Pareto-optimal public investment policy, which is obtained from the first-order condition: $\left(\frac{c_t}{\xi_t}\right)^{1/\psi} = \frac{\beta}{\phi'(\iota_t^g)} \frac{1}{\xi_t + (1-\chi_t)\partial_\chi \xi_t}$.

return can be broken down into three components:

$$
\begin{aligned}
\mathbb{E}_t[dR_t - r_t dt] = & -\mathbb{E}_t\left[dR_t \cdot \frac{d\Lambda_t}{\Lambda_t}\right] && \Longleftarrow \quad 0 = \mathbb{E}_t \frac{d(\Lambda_t \cdot R_t)}{\Lambda_t} \\
= & \mathbb{E}_t[dR_t \cdot d\chi_t] \frac{-\partial_\chi \Lambda_t}{\Lambda_t} dt && \text{(PUB risk premium)} \\
& + \mathbb{E}_t[dR_t \cdot d\sigma_t] \frac{-\partial_\sigma \Lambda_t}{\Lambda_t} dt && \text{(uncertainty risk premium)} \\
& + \mathbb{E}_t[dR_t \cdot dK_t/K_t] \frac{-K_t\, \partial_K \Lambda_t}{\Lambda_t} dt && \text{(productivity risk premium)}
\end{aligned}
\tag{16}
$$

The first component captures the *PUB risk premium* that stems from the over- or undersupply of public sector capital. Intuitively, if public sector capital is undersupplied, then PUB shocks would push the capital allocation *toward* optimum, thereby decreasing the agent's marginal utility (that is, $\partial_\chi \Lambda_t < 0$). In this case, an asset with higher loadings on PUB shocks (that is, higher $\mathbb{E}_t[dR_t \cdot d\chi_t]$) would be considered risky and thus have to deliver a higher risk premium as compensation. The second and third components capture the uncertainty and productivity risk premiums, respectively. They arise because both uncertainty shocks and aggregate technology shocks are drivers of the agent's utility. In sum, this equilibrium pricing kernel Λ_t implies a three-factor structure that embeds PUB shocks, uncertainty shocks, and general economic growth shocks.

2 Model Implications

In this section, I analyze the equilibrium behavior of the model and discuss its *main* implications. In a nutshell, the model demonstrates that (1) the supply of public sector capital affects the agent's utility; (2) the price of risk for PUB shocks changes sign when public sector capital becomes over- or under-supplied; (3) the public sector investment share is positively correlated with its capital share; (4) the Pareto-optimal public investment policy dictates a higher public sector investment rate when public sector capital is undersupplied.

Value function. For a given amount of aggregate capital K_t, the agent's value function, as shown in (9), is driven by ξ_t. As meontioned before, one can interpret ξ_t as a welfare multiplier that reflects the agent's perception of future economic prospects: good (bad) economic prospects correspond to a higher (lower) ξ_t. Panel (a) in Figure 3 plots ξ as a function of the public sector capital share χ. One can see that ξ is hump-shaped with respect to χ, meaning the agent considers economic prospects to be better when the public sector capital share is neither too high nor too low. This property mainly stems from the assumption that private and public sector capital bear a certain degree of complementarity in the aggregate production. As a result of this assumption, the maximum production is achieved when the supply of public sector capital is at an optimal level with its marginal product equal to that of private sector capital; any deviation from this level (e.g., having too much or too little public sector capital) would lead to lower output for a given amount of aggregate capital. In addition, varying uncertainty can also affect the agent's perception of economic prospects and alter her preferred level of public sector capital. In particular, higher uncertainty would hurt economic prospects and increase the agent's demand for public sector capital. In any case, from the agent's perspective, public sector capital is undersupplied when $\partial_\chi \xi_t > 0$, and oversupplied when $\partial_\chi \xi_t < 0$.

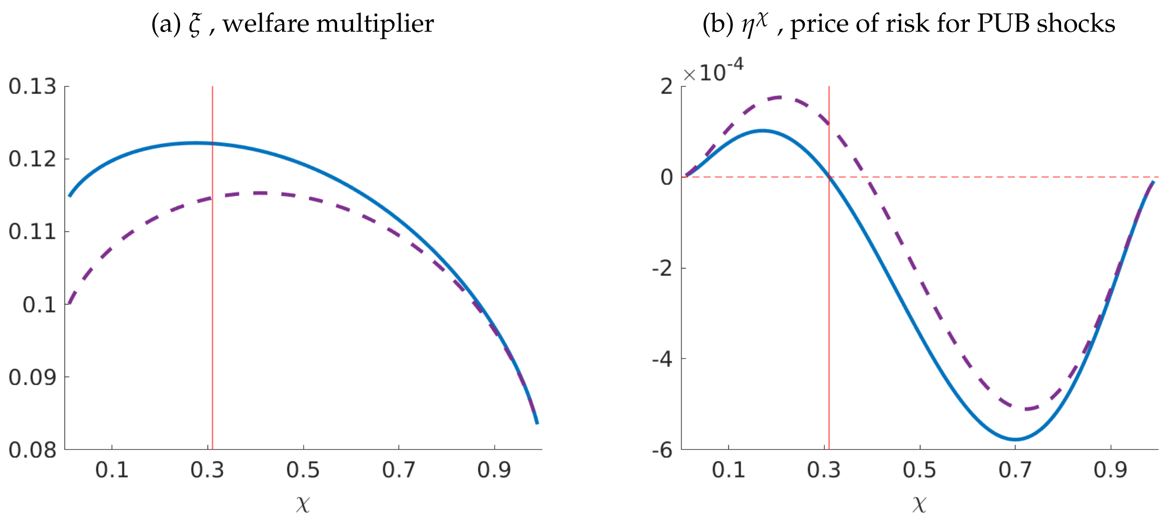


Figure 3: **Value function and the price of risk for PUB shocks.** This figure plots against the public sector capital share (χ) the welfare multiplier (ξ) and the price of risk for PUB shocks (η^χ) while holding the level of uncertainty (σ) at $\bar{\sigma}$ (solid line), and 0.05 (dashed line). The solid vertical line indicates the steady-state value of χ.

Price of risk for PUB shocks. Knowing the property of ξ, it becomes easier to understand the behavior of the risk prices. In particular, panel (b) in Figure 3 plots η_t^χ, the price of risk for PUB shocks, as a function of the public sector capital share χ. Clearly, η^χ turns positive (negative) when χ becomes too low (high). This change-of-sign behavior mainly stems from the property of ξ. To illustrate, I repeat the expression for η_t^χ here:

$$\eta_t^\chi = \left[(\gamma - 1/\psi) \frac{\partial_\chi \xi_t}{\xi_t} + 1/\psi \frac{\partial_\chi c_t}{c_t} \right] \sigma_{\chi,t}.$$

Under the baseline calibration (i.e., $\gamma = 9$ and $\psi = 2$), the sign of η^{χ} is primarily determined by $\partial_{\chi}\xi_t$: loosely speaking, when public sector capital is undersupplied ($\partial_{\chi}\xi_t > 0$), the price of risk for PUB shocks (η^{χ}) becomes positive, and vice versa. The intuition is as follows. When public sector capital is undersupplied, a PUB shock, which expands the share of public sector capital (χ), would lead to *better* economic prospects as perceived by the agent. So in this case, assets with high loadings on PUB shocks are considered risky and have to provide higher risk premia. When public sector capital is oversupplied, however, a PUB shock would lead to *worse* economic prospects. Assets with high loadings on PUB shocks, in this case, provide valuable hedges and hence should have lower risk premia.

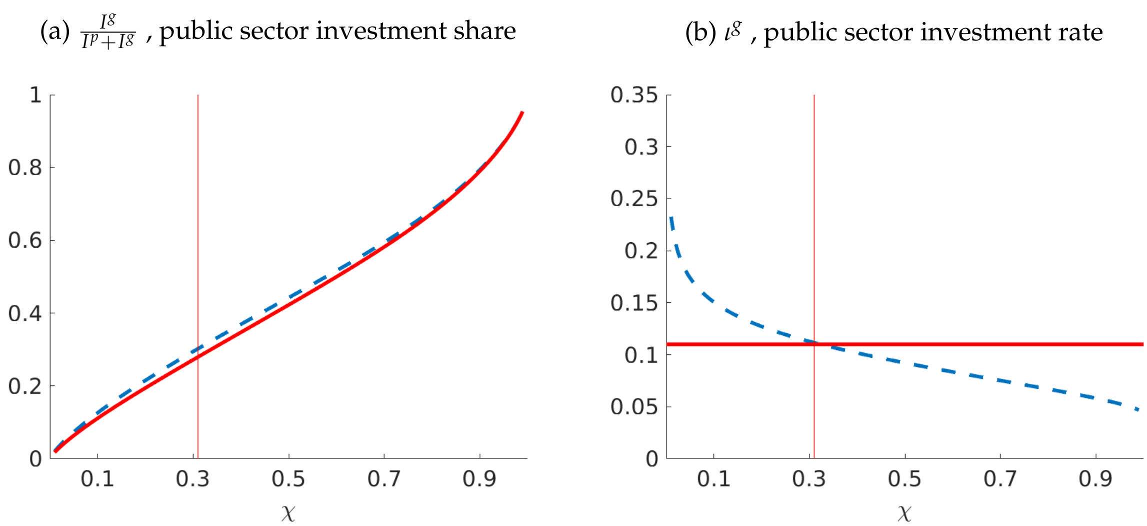


Figure 4: **Public sector investment.** This figure plots against the public sector capital share (χ) the public sector investment share ($\frac{I^g}{I^p+I^g}$) and the public sector investment rate (ι^g). The model is solved under the constant public investment rule (solid line) as well as the Pareto-optimal rule (dashed line). The solid vertical line indicates the steady-state value of χ.

Public sector investment. Panel (a) in Figure 4 displays the public sector investment share ($\frac{I^g}{I^p+I^g}$), which is positively correlated with the share of public sector capital. This is mainly driven by capital adjustment costs, which tie the movements of these two ratios together. This property underpins my empirical investigation in which I use innovations in the public sector investment share as a proxy for shocks to the share of public sector capital. Finally, panel (b) in Figure 4 compares the constant public investment rule with the Pareto-optimal rule. Clearly, the Pareto-optimal rule dictates a higher (lower) public sector investment rate when public sector capital is undersupplied (oversupplied). So in the context of my model, welfare can be improved if the government varies its investment policy in response to changing economic conditions, targeting a higher (lower) expected growth of public investment when the supply of public sector capital is too low (high).

3 Empirical Investigation: Regression-Based Approach

In this section, I empirically investigate whether and how PUB shocks are priced. The GE theory developed above has demonstrated how the share of public sector capital may enter the pricing kernel and thus become a risk factor relevant to asset pricing. Guided by this theory, I propose a three-factor asset pricing model with PUB shocks, uncertainty shocks, and the market excess return as risk factors; they represent innovations to those three state variables that govern the GE pricing kernel (13). In what follows I confront this factor model with a variety of test assets.

3.1 Primary variables and risk factors

I start by defining main variables and explaining the construction of risk factors. Other variables are introduced later when they enter my analysis.

Investment. The measurements of private and public sector investments come from the National Income and Product Accounts (NIPA) data provided by the U.S. Bureau of Economic Analysis (BEA). I follow Belo and Yu (2013) in defining *private sector investment* as the seasonally adjusted private fixed nonresidential investment (NIPA: Table 1.1.5, line 9), and *public sector investment* as the seasonally adjusted government nondefense investment (NIPA: Table 3.9.5, line 3 *minus* line 19). I define *national investment* as the sum of private and public sector investments per Aschauer (1989a), and the *public sector investment share* as the ratio of public sector investment to national investment. All variables are in real terms (deflated by corresponding price indexes) with quarterly observations that span the period 1947Q1 to 2018Q4.

Economic uncertainty. The measure of economic uncertainty is from Jurado, Ludvigson, and Ng (2015). They construct comprehensive and model-free macroeconomic uncertainty indexes that capture the common variation in uncertainty among a variety of economic indicators. [20] This measure is well-suited for the study of aggregate uncer-

[20] Specifically, Jurado, Ludvigson, and Ng (2015) define individual uncertainty as the conditional volatility of the forecast error for each indicator. They estimate the forecast error by fitting a diffusion index model to the time series of these indicators. Then, with the estimated forecast error, they infer its con-

tainty and its comovement with other variables. I pick their 1-month-ahead macro uncertainty index and aggregate it to a quarterly frequency (by simple average). The resulting measure spans the period 1960Q3 to 2018Q4.

It is worth mentioning that economic uncertainty is difficult to quantify. Researchers have taken various approaches to measure it, resulting in a variety of uncertainty indicators yet little consensus on which one is the best (Caldara et al., 2016). The only agreement on this matter is probably that uncertainty is countercyclical (Bloom, 2014). That said, I choose the JLN measure for good reason: it has relatively long sample period and also possesses more predictive content than other measures. [21]

Constructing risk factors. Figure 1 plots the public sector investment share as well as the JLN uncertainty index. From these two variables, I construct two risk factors, denoted by $PubFac$ and $UncFac$, as shocks to the public sector capital share (PUB shocks) and economic uncertainty, respectively. They are defined as innovations in the AR(1) representation of the public sector investment share and the JLN uncertainty index. For convenience, I standardize $PubFac$ and $UncFac$ to unit variance. Together with the market excess return, these factors constitute the three-factor model that underpins my subsequent analysis.

Figure 5 displays the time series of $PubFac$ and $UncFac$. Both factors seem countercyclical because they often witness sizeable positive spikes during recessions. Most notably, in the Great Recession, $UncFac$ reached its nadir at the height of the crisis. $PubFac$ also showed a big increase, especially at the passage of the American Recovery and Reinvestment Act (ARRA), a fiscal stimulus bill that includes large public sector investment.

Table 1 documents the correlations of $PubFac$ and $UncFac$ with a selection of economic indicators. Both $PubFac$ and $UncFac$ are negatively related to GDP growth and positively related to changes in the unemployment rate, confirming the countercyclicality of the public sector investment share and economic uncertainty. $PubFac$ positively correlates with government consumption and the fiscal deficit (relative to GDP), suggesting that a higher public sector investment share tends to coincide with increased government consumption and a larger deficit.

ditional volatility using a stochastic volatility model. The final products, the macroeconomic uncertainty indexes, are constructed by aggregating together these individual uncertainty measures.

[21] Caldara, Fuentes-Albero, Gilchrist, and Zakrajsek (2016) conduct a "horse race" exercise, demonstrating that the JLN measure is more informative about future economic activity.

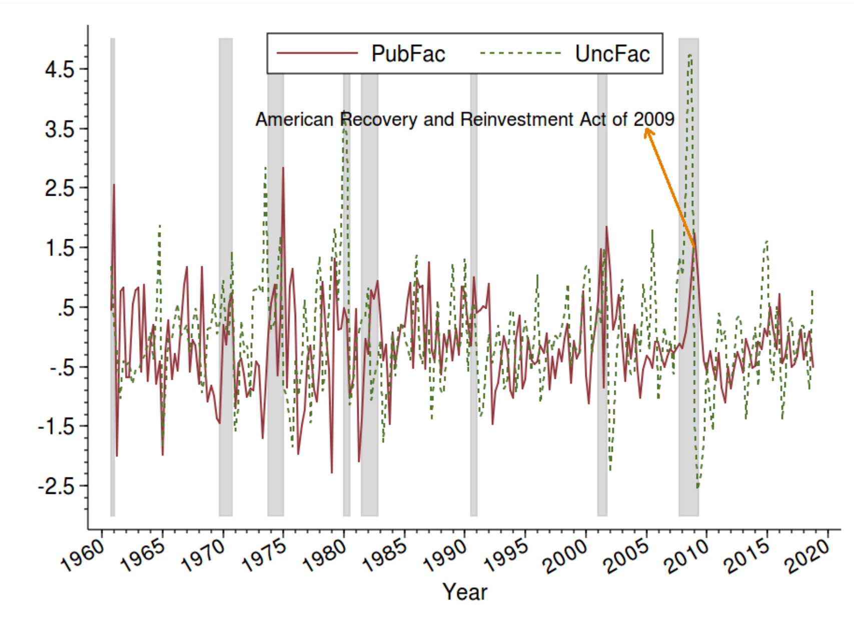


Figure 5: **Risk factors.** This figure plots two risk factors denoted by *PubFac* and *UncFac*, which are defined as innovations in the AR(1) representations of the public sector investment share and economic uncertainty, respectively. *PubFac* and *UncFac* are standardized to unit variance. Shaded areas indicate U.S. recessions defined by NBER.

3.2 Empirical approach

To examine the asset pricing role of PUB shocks, I follow a standard two-pass regression approach. The first pass estimates the betas (that is, exposure to risks) for each test asset i via a time-series regression of the asset's excess returns, $r^e_{i,t}$, on the risk factors:

$$r^e_{i,t} = a_i + \boldsymbol{f}'_t \boldsymbol{\beta}_i + \xi_{i,t} \ , \ t = 1, ..., T$$

where $\boldsymbol{f}$ is a vector of risk factors, and $\boldsymbol{\beta}_i$ is a vector of betas (to be estimated) for asset i. The second pass estimates the risk prices via a cross-sectional regression of assets' (time-series) average excess returns on their estimated betas:

$$\overline{r^e_i} = \alpha + \boldsymbol{\beta}'_i \boldsymbol{\lambda} + \epsilon_i \ , \ i = 1, ..., N$$

where $\overline{r^e_i}$ is the unconditional mean excess return for asset i, $\boldsymbol{\beta}_i$ denotes the estimated betas from the first pass, and $\boldsymbol{\lambda}$ is a vector of risk prices to be estimated. My primary factor model consists of *PubFac*, *UncFac*, and the market excess return ($\boldsymbol{f} = [PubFac$, $UncFac$, $MktRf]$), while I also consider the Fama and French (1993) model ($\boldsymbol{f} = [SMB$, HML, $MktRf]$) as a comparison.

Table 1: **Risk factors' correlations with common economic indicators.** This table presents the correlations of two risk factors, *PubFac* and *UncFac*—which are defined as innovations in the AR(1) representations of the public sector investment share and economic uncertainty, respectively— with a selection of economic indicators including: the market excess return (from Ken French's website); the growth (log change) of GDP (NIPA: Table 1.1.5, line 1) and government nondefense consumption (NIPA: Table 3.9.5, line 2 *minus* line 18), both of which are in real terms; and the changes in civilian unemployment rate (from FRED) and the deficit-to-GDP ratio (NIPA: Table 3.1, (-) line 43 *to* GDP).

	PubFac	*UncFac*
PubFac	1.00	
UncFac	0.00	1.00
Market excess return	0.16	-0.23
GDP (log change)	-0.16	-0.37
Unemployment rate (change)	0.36	0.20
Govt. consumption (log change)	0.13	0.06
Deficit/GDP (change)	0.26	0.14

This regression approach is standard and widely commended for its transparency, but like any other approach, it has limitations. A well-known one is that betas are estimated via time-series regressions and hence are inaccurate by definition. This is particularly relevant when nontraded factors are used (as is the case here), because, if a nontraded factor contains substantial noise, the estimated betas will be understated while the corresponding risk prices overstated. To assess the extent to which this limitation bites, I use Shanken (1992)'s correction to adjust standard errors, checking if it makes a big difference. Another limitation is that implicit in this approach is a presumption of constant betas for each asset, whereby the estimated $\boldsymbol{\lambda}$ gives the time-series averages of risk prices. One can reasonably argue that this presumption is untenable, but relaxing it requires more sophisticated estimators or granular data; both seem beyond reach at this point. So I leave for future research the exploration of alternative approaches.

Test assets. For test assets I consider a wide range of standard equity portfolios formed on size, BM, momentum, investment, and profitability. These portfolios are known to exhibit sizeable differences in average returns (Fama and French, 2015). [22] Besides, I also consider portfolios formed on past exposure to *PubFac*. Specifically, at each quarter end, I sort stocks in the Center for Research in Security Prices (CRSP) database [23] by their past exposure to *PubFac* (or β_{Pub}) and then stratify them into decile portfolios. I

[22] I do not consider other asset classes like corporate bonds and derivatives because, according to Adrian, Etula, and Muir (2014) and He, Kelly, and Manela (2017), financial intermediaries tend to be the marginal investors in these more sophisticated asset markets rather than households.

[23] I only include stocks with share codes 10 or 11 and listed on the NYSE, AMEX, or NASDAQ.

obtain the pre-formation β_{Pub} for each stock via a rolling regression of its excess returns on $PubFac$, $UncFac$, and the market excess return with a 40-quarter trailing window (I require at least 32 quarters of data); the pre-formation β_{Pub} is measured by the coefficient on $PubFac$. These portfolios are rebalanced every quarter, and their returns are computed as the value-weighted averages of their constituent stocks' returns.

3.3 Results

I start by pricing 25 size and value sorted portfolios with my primary factor model, comparing it with the Fama and French (1993) model; Table 2 presents the results. Panel (a) reports the mean excess returns and the estimated betas for all portfolios. Consistent with the literature, average return generally falls from small stocks to big stocks while rises from growth stocks to value stocks. As for betas, an interesting observation is that exposure to PUB shocks seems to negatively correlate with size: small stocks tend to be more sensitive to variations in the public sector investment share. Similar patterns can be found in almost every investment, profitability, and momentum quintile, as shown in Table B.4. This implies that augmenting public sector capital is likely to benefit small firms more than big firms. [24]

Panel (b) reports the estimated risk prices and several test diagnostics. The price of risk for PUB shocks (λ_{Pub}), which is my main focus, is positive and statistically significant. The t-statistics, whether based on Fama and MacBeth (1973) standard errors adjusted for autocorrelation or ordinary least squares (OLS) standard errors adjusted for beta estimation errors per Shanken (1992), are both above 2. The economic magnitude of λ_{Pub} is also sizeable at 1.06% per quarter. With PUB betas ranging from -0.43 to 0.95 for this group of assets, this amounts to a roughly $6(\approx 1.38 \times 1.06 \times 4)$ percent differential in expected annual returns. (As a reference point, the range of the mean excess returns across these assets is about 9 percent per year.) This result points to PUB shocks as good news from investors' perspective, as they demand higher returns from assets that load more positively on PUB shocks. Thus an expansion in public sector investment (relative to private) is likely to accompany a favorable shift in investors' welfare.

The pricing performance of my factor model is modestly strong. The mean absolute pricing error (MAPE) is low at 0.28% per quarter, while the adjusted R^2 is moderate at

[24] Anecdotal evidence also supports the idea that small firms may benefit more from greater public sector investment. A good example is the construction industry, an undoubted beneficiary that has "the largest small business concentration of any industry" (Mills, 2014). According to *The Economist* (2017), the construction industry has highly fragmented structure: "less than 5% of builders work for construction firms that employ over 10,000 workers."

51%. The χ^2 statistic is at a particularly low level of 20.50, indicating that the hypothesis of zero joint pricing errors across assets is not rejected. These statistics are close to that for the Fama and French (1993) model reported in panel (c), which is pretty impressive given the fact that the Fama and French (1993) model is statistically tailored to price this cross section while my factor model is theoretically motivated. However, I do not want to stretch too far because the estimation also reveals a large intercept (α) that indicates a certain degree of misspecification. (The same problem attends the Fama-French model.) So I conduct more tests to check the robustness of these findings.

Robustness: other assets. Next, I confront my primary model with more test assets and see how it fares. The results, reported in Table 3, echo and strengthen the previous findings. The risk price for $PubFac$ remains positive and statistically significant across different sets of test assets. This is true even when all portfolios are included in the tests. Interestingly, in an unreported result, I find in this larger cross section that my primary model provides a better fit (in terms of higher R^2) relative to the Fama and French (1993) model. This finding is also mirrored in Figure 6, which plots the realized mean excess returns on all portfolios against their model-implied counterparts. When priced by my primary model, these assets line up closer to the 45-degree line.

(a) Primary model (b) Fama and French (1993) model

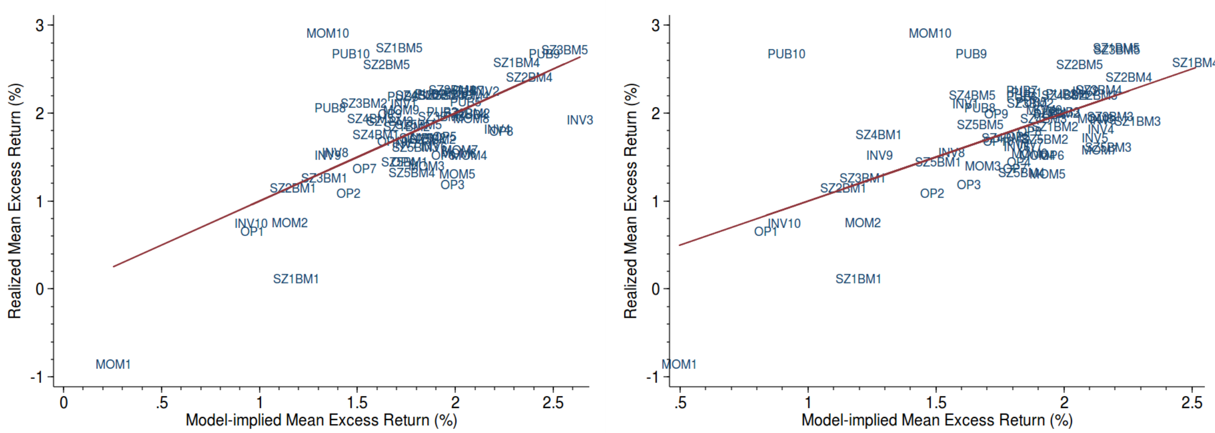


Figure 6: **Realized versus model-implied mean excess returns.** This figure compares the realized versus the model-implied mean excess returns for all test assets including 25 size and value sorted portfolios, 10 β_{Pub} sorted portfolios, 10 momentum portfolios, 10 investment portfolios, and 10 profitability portfolios. Two factor models are considered: the primary model displayed in panel (a) consists of $PubFac$, $UncFac$, and $MktRf$; the Fama and French (1993) model displayed in panel (b) consists of SMB, HML, and $MktRf$. The sample is quarterly and spans the period 1969Q1 to 2018Q4.

In summary, a theoretically founded factor model that includes $PubFac$, $UncFac$, and the market excess return performs fairly well in pricing a wide range of standard equity portfolios. The estimated risk price for PUB shocks is consistently positive and significant. This finding suggests that increases in the share of public sector investment tend to concur with better welfare for investors.

Table 2: **Two-pass asset pricing analysis: 25 Size-BM equity portfolios.** This table presents the results of a two-pass asset pricing analysis. Panel (a) reports the test assets' mean quarterly excess returns ($\overline{r_i^e}$) and estimated betas. The latter are obtained by running a time-series regression specified as $r_{i,t}^e = a_i + \boldsymbol{f}_t' \boldsymbol{\beta}_i + \xi_{i,t}$ for each asset i, where $r_{i,t}^e$ is the asset's excess return, $\boldsymbol{f}_t$ represents a vector of risk factors, and $\boldsymbol{\beta}_i$ denotes a vector of beta estimates. Panel (b) reports the risk prices estimated from a cross-sectional regression of test assets' mean excess returns on estimated betas, that is, $\overline{r_i^e} = \alpha + \boldsymbol{\beta}_i' \boldsymbol{\lambda} + \epsilon_i$. The t-statistics are based on either Fama and MacBeth (1973) standard errors with Newey and West (1987) correction (one lag) or ordinary least squares (OLS) standard errors with Shanken (1992) correction. Also reported are test diagnostics including mean absolute pricing error (MAPE), adjusted R^2, and a χ^2 statistic along with the p-value that tests whether the pricing errors are jointly zero. The primary factor model comprises *PubFac*, *UncFac* and the market excess return. The test assets include 25 size and value sorted equity portfolios. The sample is quarterly and spans the period 1960Q4 to 2018Q4. As a comparison, panel (c) reports the analogous statistics for the Fama and French (1993) model.

(a) Mean excess returns and betas by asset

		Size									
		Small				Big	Small				Big
		$\overline{r_i^e}$					β_{Pub}				
	Growth	0.89	1.41	1.45	1.83	1.50	0.63	0.43	0.32	0.33	-0.02
		2.25	2.26	2.34	1.83	1.64	0.77	0.95	0.13	0.26	-0.04
BM		2.26	2.52	2.16	2.11	1.71	0.54	0.52	0.20	0.01	-0.43
		2.93	2.71	2.57	2.51	1.50	0.86	0.93	0.38	0.26	0.01
	Value	3.20	2.93	2.95	2.48	2.05	0.68	0.23	0.83	0.50	-0.38
		β_{Unc}					β_{Mkt}				
	Growth	0.50	0.52	0.30	0.50	-0.06	1.57	1.49	1.37	1.27	1.00
		0.06	0.03	-0.19	-0.16	0.15	1.32	1.21	1.13	1.07	0.90
BM		-0.49	-0.28	-0.11	-0.54	-0.16	1.16	1.09	1.01	0.99	0.79
		-0.07	-0.22	-0.38	-0.30	-0.70	1.08	1.03	0.98	0.96	0.85
	Value	-0.84	-0.27	0.15	-0.77	0.35	1.14	1.11	1.02	1.07	0.95

(b) Risk prices and test diagnostics

	λ_{Pub}	λ_{Unc}	λ_{Mkt}	α	Test diagnostics			
Coefficient	1.06	-0.35	-1.61	3.52	MAPE	0.28	χ^2	20.50
[t-FMNW]	[3.80]	[-1.55]	[-1.61]	[3.98]	Adj. R^2	0.51	p-value	0.55
[t-Shanken]	[2.16]	[-0.92]	[-1.03]	[2.33]				

(c) Comparison with the Fama and French (1993) model

	λ_{SMB}	λ_{HML}	λ_{Mkt}	α	Test diagnostics			
Coefficient	0.39	1.06	-1.74	3.41	MAPE	0.22	χ^2	58.12
[t-FMNW]	[1.08]	[2.64]	[-1.67]	[3.71]	Adj. R^2	0.67	p-value	0.00
[t-Shanken]	[1.08]	[2.83]	[-1.61]	[3.65]				

Table 3: **Two-pass asset pricing analysis: other portfolios.** This table presents the results of a two-pass asset pricing analysis. The procedure and relevant statistics are described in more detail in Table 2. Panel (a) summarizes the test assets' mean (quarterly) excess returns and estimated betas. $\mu[\cdot]$ and $\sigma[\cdot]$ denote the cross-sectional mean and standard deviation, respectively. Panel (b) reports the estimated risk prices. The factor model comprises $PubFac$, $UncFac$ and the market excess return. The test assets are 25 size and value sorted equity portfolios (Column 1) plus 10 β_{Pub} sorted portfolios (Column 2), or 10 momentum portfolios (Column 3), or 10 investment portfolios (Column 4), or 10 profitability portfolios (Column 5), or all 65 portfolios together (Column 6). The sample is quarterly and spans the period 1969Q1 to 2018Q4; the start is dictated by the β_{Pub} portfolios.

(a) Mean excess returns and betas by asset

	SZBM25	PUB10	MOM10	INV10	OP10	All
$\mu[\overline{r^e}]$	2.16	2.24	1.41	1.69	1.45	1.74
$\sigma[\overline{r^e}]$	0.58	0.24	0.97	0.41	0.39	0.63
$\mu[\beta_{Pub}]$	0.36	0.21	0.16	0.06	-0.01	0.25
$\sigma[\beta_{Pub}]$	0.38	0.40	0.39	0.40	0.24	0.46
$\mu[\beta_{Unc}]$	-0.12	-0.03	-0.25	-0.07	0.00	-0.08
$\sigma[\beta_{Unc}]$	0.38	0.30	0.51	0.15	0.22	0.37
$\mu[\beta_{Mkt}]$	1.10	0.99	1.04	0.99	1.02	1.05
$\sigma[\beta_{Mkt}]$	0.19	0.12	0.20	0.16	0.13	0.17
$\mu[R^2]$	0.77	0.85	0.81	0.88	0.90	0.83
Quarters	200	200	200	200	200	200

(b) Risk prices and test diagnostics

	SZBM25	SZBM25 + PUB10	SZBM25 + MOM10	SZBM25 + INV10	SZBM25 + OP10	All
λ_{Pub}	0.81	0.67	0.80	0.72	0.82	0.67
[t-FMNW]	[3.18]	[3.04]	[2.83]	[2.81]	[3.14]	[2.88]
[t-Shanken]	[2.11]	[2.22]	[1.79]	[1.96]	[2.09]	[2.03]
λ_{Unc}	0.01	0.07	0.64	-0.17	-0.09	0.51
[t-FMNW]	[0.05]	[0.27]	[2.37]	[-0.86]	[-0.44]	[2.12]
[t-Shanken]	[0.04]	[0.21]	[1.70]	[-0.66]	[-0.32]	[1.66]
λ_{Mkt}	-2.59	-2.31	-3.14	-2.09	-2.37	-2.60
[t-FMNW]	[-2.32]	[-2.27]	[-3.00]	[-2.04]	[-2.28]	[-2.72]
[t-Shanken]	[-1.74]	[-1.83]	[-2.15]	[-1.61]	[-1.69]	[-2.14]
α	4.35	4.20	4.92	3.79	4.02	4.36
[t-FMNW]	[4.38]	[5.01]	[5.59]	[4.45]	[4.64]	[5.84]
[t-Shanken]	[3.02]	[3.75]	[3.59]	[3.19]	[3.11]	[4.11]
MAPE	0.25	0.26	0.38	0.24	0.26	0.36
Adj. R^2	0.56	0.39	0.42	0.59	0.57	0.38
χ^2	28.43	74.51	44.53	38.59	38.81	160.71
p-value	0.16	0.00	0.07	0.20	0.19	0.00

4 Empirical Investigation: Portfolio-Based Approach

The regression-based approach has its limitations (as already mentioned) that might raise concerns about the validity of its results. So in this section, I provide additional evidence via a portfolio-based approach using a sample of U.S. government contractors. The idea is as follows. I postulate that firms with heavier reliance on sales to the U.S. government load more positively on PUB shocks. Thereby if the price of risk for PUB shocks is positive, high-dependency firms should carry higher risk premiums compared to low-dependency firms. This is exactly what I find.

4.1 Sample construction and portfolio formation

From the CRSP/Compustat Merged (CCM) database, I collect a sample of U.S. government contractors. Using their stocks I form portfolios based on the extent of their dependency on government customers for revenue.

Identifying government contractors. The *Financial Accounting Standards Board* (1997) requires firms to report their sales to major customers including the U.S. government (federal, state, and local). [25] This information, which is in the Compustat Customer Segment file, together with other accounting information from the Compustat Fundamental Annual file allows me to compute for each firm-year the fraction of sales accounted for by government customers (denoted by StG). Every year I define government contractors as firms that reported positive sales to government at least once over the past three years. I exclude firms in the healthcare and pharmaceutical industries, the consumer goods and services industries as well as the defense industry, because transactions between these firms and government, if any, are more likely to stem from other types of government spending than public sector investment. For example, healthcare and pharmaceutical companies have business connections with government mainly because of their involvements in social security programs such as Medicaid and Medicare (Goldman, 2019). Government purchases from consumer goods and services firms are more likely to be categorized as government consumption rather than investment. As for firms in the defense industry, their transactions with government apparently come from defense spending.

[25] The *Financial Accounting Standards Board* (1997) dictates that "an enterprise shall provide information about the extent of its reliance on its major customers. If revenues from transactions with a single external customer amount to 10 percent or more of an enterprise's revenues, the enterprise shall disclose that fact, the total amount of revenues from each such customer ... For purposes of this Statement, ... the federal government, a state government, a local government (for example, a county or municipality), or a foreign government each shall be considered as a single customer." (para. 39)

After this exclusion (and other standard filters), I find 1,242 government contractors with 9,944 firm-year observations spanning 1980 to 2017; these firms are mainly from the construction and manufacturing industries (with SIC between 1500 and 3999). [26]

Panel (a) in Table 4 provides summary statistics for this sample of government contractors. As shown, there is substantial variation in StG. The median government contractor has 18.5% of its sales generated by government customers. About a quarter of government contractors derive more (less) than 45% (5%) of their sales revenue from government. For a tenth of government contractors, sales to government account for more than 75% of their total sales. Regarding other firm characteristics, the average government contractor has a book-to-market ratio of 0.72 and market leverage of 0.21; its book value of assets (total sales) grows 14.7% (14.1%) year-on-year; its profitability ratio and return on assets are 0.16 and 0.3%, respectively. These numbers are similar to those in Goldman (2019), who reported, for a sample of government contractors in 2005 and 2006, an average sales growth of 19%, return on assets of -3.1%, and leverage of 0.23.

Forming portfolios on government dependency. Using these government contractors' stocks (which are ordinary common shares listed on the NYSE, AMEX, or NASDAQ), I form portfolios based on the extent to which they depend on government customers for revenue. Every year I measure a firm's government dependency by $\overline{StG}_{-2,0}$, a three-year trailing average of StG. [27] Following the convention in the literature, I form stock portfolios at the end of June in each year t based on the quintiles of government dependency computed for the previous year (that is, $\overline{StG}_{t-3\to t-1}$). I also consider a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. These portfolios are held from July of year t to June of year $t+1$, by which time the next formation happens. The first set of portfolios were formed in 1981, and the last in 2018.

Panel (b) in Table 4 compares firms in different government dependency portfolios. Unsurprisingly, high-dependency firms tend to have high StG in the year before formation. In other aspects, however, firms are similar across portfolios. Although firms with the highest dependency are somewhat smaller and have slightly lower leverage and higher asset growth and operating profitability compared to firms with the lowest depen-

[26] Appendix B provides more details on the sample construction.

[27] I choose this moving-average measure for good reason. First, a firm only needs to report its sales to government customers when that accounts for more than 10% of its total sales in a fiscal year. For years with no reported sales to government, StG is zero though the real value can be larger than that. Also, there are some data errors as noted by Goldman (2019). For example, occasionally foreign governments are mistaken for the U.S. government, and the U.S. government agencies are mistaken for private companies. Using a moving average can help smooth out, at least in part, some of these data omissions and errors.

dency, the differences are minor. This is confirmed by Figure B.1, which uses box plots to compare the distributional properties of firm characteristics across portfolios; it shows that other firm characteristics are not systematically related to government dependency.

4.2 Portfolio analysis

Given these government dependency portfolios, I first establish the link between government dependency and exposure to public sector investment. Then I infer investors' opinion on public sector investment by comparing the average returns on different dependency portfolios.

Is government dependency a relevant proxy? I hypothesize that the extent of a firm's dependency on government is a relevant proxy for its exposure to changes in public sector investment. Now I provide support for this hypothesis. First, I show that government dependency is persistent. Specifically, I examine whether past dependency predicts future dependency via a predictive regression specified as

$$StG_{i,t+h} = \alpha_h + \beta_h \, \overline{StG}_{i,t-2\to t} + \epsilon_{i,t+h} \tag{17}$$

where h is the forecast horizon, and $\overline{StG}_{i,t-2\to t}$ is the average fraction of sales to government over the past three years ending in year t. If government dependency is persistent, then β_h would be positive and close to one. This is exactly the case. As shown in Table 5, at the one-year horizon, a one percentage point increase in $\overline{StG}_{-2,0}$ is associated with a 0.93 percentage point increase in StG; this figure remains high at 0.86 even for the three-year horizon. It suggests that a firm with high government dependency in the past also tends to have a large fraction of sales contributed by government in the near future.

Second, I show that high-dependency firms are more sensitive to changes in public sector investment. I examine the relation between firms' performance and public sector investment, and more importantly, whether the magnitude of this relation is greater for high-dependency firms. I consider the following regression

$$\nabla[sales/earnings]_{i,t+1} = \alpha + \beta_1 \, \overline{StG}_{i,t-2\to t} + \beta_2 \, \nabla i^g_{t+1} + \beta_3 \, \nabla i^g_{t+1} \times \overline{StG}_{i,t-2\to t} + \epsilon_{t+1} \tag{18}$$

where $\nabla[sales/earnings]_{i,t+1}$ is the sales or earnings (EBITDA) growth for firm i in year $t+1$, and ∇i^g_{t+1} is the contemporaneous public sector investment growth. The last two columns of Table 5 report the results. To understand, consider two average firms: one from the lowest dependency quintile and another from the highest. The estimated coefficients indicate that, for the former ($\overline{StG}_{-2,0} = 0.03$), a one percentage point increase in the growth rate of public sector investment accompanies a 0.29 (0.10) percentage point increase in its sales (earnings) growth; whereas for the latter ($\overline{StG}_{-2,0} = 0.74$), the same increase in public sector investment growth is associated with a 1.01 (0.87) percentage point increase in its sales (earnings) growth. [28] It clearly suggests that firms with higher government dependency are more sensitive to variations in public sector investment. Later, I also show that high-dependency portfolios have higher β_{Pub}, which again supports that government dependency is a relevant proxy.

Comparing returns on government dependency portfolios Having established the link between government dependency and exposure to public sector investment for this sample of government contractors, I then turn to examining the average returns on dependency portfolios. I obtain stock-level data from the Center for Research in Security Prices (CRSP) Monthly Stock file. [29] I consider both value- and equal-weighted portfolios.

Panel (a) of Table 6 reports the mean excess returns along with the Sharpe ratios and β_{Pub} for value-weighted portfolios over the full sample period (1981-2018). One can see that stocks in high-dependency portfolios tend to provide higher average returns. The long-short portfolio (long the highest-dependency portfolio and short the lowest-dependency portfolio) delivers an average return of 0.62% per month (that is, 7.43% per year) and has a Sharpe ratio (annualized) of 36.14%. Moreover, this return pattern line up well with the differences in β_{Pub}. Using the estimated price of risk for PUB shocks (λ_{Pub}) from Section 3, the spread in β_{Pub} between the highest- and lowest-dependency portfolio translates to a return spread of about 8.55% ($\approx 3.19 \times 0.67 \times 4$) per year.

A similar pattern emerges from panel (a) of Table 7, where I consider equal-weighted portfolios; it also reveals a positive relation between government dependency and average return. The long-short portfolio provides an average return of 0.35% per month (that is, 4.21% per year) and has a Sharpe ratio of 31.70%. The difference in β_{Pub} between the highest- and lowest-dependency portfolio translates to a return spread of 4.29% ($\approx 1.60 \times 0.67 \times 4$) per year. These dependency patterns in average returns are graphically shown in Figure 7.

Given these sizable spreads in average returns, a natural question is whether they are driven by government contractors' differential loadings on classic risk factors regardless

[28]The results for earnings growth are not statistically significant at conventional levels, which may be caused by the fact that earnings growth is much more noisy than sales growth: there are a lot more instances of missing or negative values for EBITDA than for sales.

[29]Monthly stock returns are corrected for delisting (Shumway, 1997) and winsorized at 1st and 99th percentiles. But these adjustments make little difference.

of their exposure to public sector investment. I address this question by estimating the portfolio alphas with respect to a set of standard risk factors in the literature, including the market factor (MKT), the size factor (SMB), and the value factor (HML) from Fama and French (1993) as well as the momentum factor (MOM) from Carhart (1997) and the liquidity factor (LIQ) from Pastor and Stambaugh (2003). The results for value-weighted portfolios are shown in panel (b) of Table 6; it confirms that the spread in average returns between high- and low-dependency firms is not accounted for by loadings on these risk factors. The long-short portfolio's alpha is 0.82% monthly with a t-statistic of 2.42. For equal-weighted portfolios the conclusion is the same: the dependency premium cannot be explained by exposure to classic risk factors. The long-short portfolio's alpha, shown in panel (b) of Table 7, is 0.56% monthly with a t-statistic of 2.40. Figure 7 provides a clear picture of this pattern in portfolio alphas.

Time variation in PUB risk premium. If the spread in average returns between high- and low-dependency firms is driven by a PUB risk premium, then, as the GE theory suggests, its sign and magnitude reflect investors' opinion on whether public sector capital is underinvested. If yes, high-dependency firms should provide higher expected returns compared to low-dependency firms. Following this logic, the results above seem to suggest that investors perceive an overall shortfall in public sector investment during the 1981-2018 period. But a natural question is whether this shortfall is getting better or worse over time. This question is particularly relevant because policymakers are recently considering potential increases in public investment. If, for example, the PUB risk premium was high in earlier years but had diminished in more recent years, then the case for greater public sector investment would be weakened by such observation. Nevertheless, what I find is the opposite.

I split the sample into two subperiods of equal length: 1981 to 1999 and 2000 to 2018, and repeat the analysis above for these two subperiods separately. The results are reported in Table 8 and graphically displayed in Figure 8 and 9. I find that the differences in average returns across government dependency portfolios are small for the 1981-1999 period, but they become large in the 2000-2018 period. In particular, when using equal weight, the long-short portfolio actually has a slightly negative average return of -0.03% per month for the 1981-1999 period. In comparison, for the 2000-2018 period, the long-short portfolio provides a notably higher average return: 0.87% per month (that is, 10.44% per year) when using value weight and 0.73% per month (that is, 8.76% per year) when using equal weight. And again I confirm that these return spreads cannot be explained by exposure to classic risk factors. [30] So the results of this exercise suggest that the inadequacy in public investment, if any, is minor in the 1980s and 1990s, but it has become more severe in recent years.

This finding accords with the notion that the cost of government spending was high in the 1980s and 1990s, so the net benefits of public investment were probably low. As noted by Furman and Summers (2019), the fiscal consolidation efforts at that time might have been beneficial and contributed to higher economic growth. Also, this finding is consistent with the declining trend in the public sector investment share. Intuitively, if the optimal capital allocation between the private and public sectors remains constant, then the declining share of public sector investment implies that a shortfall in public investment is more likely to exist in more recent periods. Indeed, Figure 10 demonstrates an evident *negative* correlation between the public sector investment share and the average future return (over the subsequent seven years) on the equal-weighted long-short dependency portfolio; the correlation coefficient is 0.56 and highly significant. It reveals that a lower public sector investment share tends to precede a larger return spread between high- and and low-dependency firms.

[30] Table B.2 and B.3 along with Figure B.2 show that the results remain unchanged when I include the profitability and investment factors from Fama and French (2015).

Table 4: **Summary statistics.** Panel (a) summarizes a selection of firm characteristics for a sample of U.S. government contractors. Every year government contractors are defined as firms with positive sales to government over the past three years. The reported characteristics include *StG* ratio (sales to government divided by total sales), market capitalization (in billions of 2012 dollars, deflated by GDP price index), book-to-market ratio, market leverage, asset growth, sales growth, operating profitability, and return on assets. Panel (b) compares the means of these characteristics across portfolios formed on government dependency (that is, the extent to which a firm depends on government customers for revenue). Government dependency is measured by $\overline{StG}_{-2,0}$, a three-year trailing average of StG. This government contractor sample consists of 9,944 firm-year observations spanning 1980 to 2017. The first portfolio formation was at the end of June in 1981, and it was based on government dependency computed for 1980; the same procedure are repeated every year thereafter until 2018. Detailed sample construction and variable calculations are in Appendix B.

(a) Government contractors

			Percentiles				
Characteristics	Mean	S.D.	10th	25th	50th	75th	90th
StG	0.284	0.283	0.000	0.059	0.185	0.442	0.758
Market capitalization	1.435	4.604	0.010	0.029	0.110	0.588	2.790
Book-to-market	0.715	0.539	0.187	0.345	0.590	0.935	1.391
Market leverage	0.212	0.212	0.000	0.032	0.151	0.331	0.530
Asset growth	0.147	0.386	-0.164	-0.034	0.066	0.208	0.512
Sales growth	0.141	0.358	-0.182	-0.035	0.084	0.235	0.478
Operating profitability	0.163	0.488	-0.222	0.049	0.196	0.346	0.543
Return on assets	0.003	0.180	-0.177	-0.015	0.044	0.086	0.137

(b) Government dependency portfolios

	Govt. dependency portfolios				
Characteristics	1 (low)	2	3	4	5 (high)
StG	0.030	0.099	0.197	0.371	0.726
Market capitalization	1.786	1.401	1.593	1.028	1.368
Book-to-market	0.707	0.708	0.721	0.731	0.707
Market leverage	0.220	0.216	0.227	0.197	0.201
Asset growth	0.140	0.136	0.133	0.145	0.181
Sales growth	0.147	0.152	0.128	0.130	0.149
Operating profitability	0.156	0.141	0.172	0.146	0.199
Return on assets	0.009	-0.007	0.003	-0.003	0.016

Table 5: **Government dependency is a persistent proxy for exposure to public sector investment.** This table reports the estimation results of a predictive regression: $StG_{i,t+h} = \alpha_h + \beta_h \, \overline{StG}_{i,t-2\to t} + \epsilon_{i,t+h}$, where $StG_{i,t+h}$ is the fraction of sales to government in year $t+h$ for firm i, $\overline{StG}_{i,t-2\to t}$ is the average fraction of sales to government from year $t-2$ to t, and h is the forecast horizon. It also reports the results of the following regression: $\nabla[sales/earnings]_{i,t+1} = \alpha + \beta_1 \, \overline{StG}_{i,t-2\to t} + \beta_2 \, \nabla i^g_{t+1} + \beta_3 \, \nabla i^g_{t+1} \times \overline{StG}_{i,t-2\to t} + \epsilon_{t+1}$ where $\nabla[sales/earnings]_{i,t+1}$ is the sales or earnings (EBITDA) growth for firm i in year $t+1$, and ∇i^g_{t+1} is the contemporaneous public sector investment growth. The sample consists of 9,944 firm-year observations spanning 1980 to 2017. Industries are classified by two-digit SIC code. In parentheses are robust standard errors clustered at the firm level. Attached stars (*, **, ***) indicate (1, 5, 10%) statistical significance.

	$StG_{i,t+h}$			$\nabla sales_{i,t+1}$	$\nabla earnings_{i,t+1}$
	$h=1$	$h=2$	$h=3$		
$\overline{StG}_{i,t-2\to t}$	0.93***	0.89***	0.86***	-0.05***	-0.05
	(0.01)	(0.01)	(0.02)	(0.02)	(0.04)
∇i^g_{t+1}				0.26*	0.07
				(0.15)	(0.28)
$\nabla i^g_{t+1} \times \overline{StG}_{i,t-2\to t}$				1.01***	1.08
				(0.38)	(0.70)
Fixed effects	Year	Year	Year	Industry	Industry

(a) Mean excess returns (v.w.) (b) Five-factor alphas (v.w.)

(c) Mean excess returns (e.w.) (d) Five-factor alphas (e.w.)

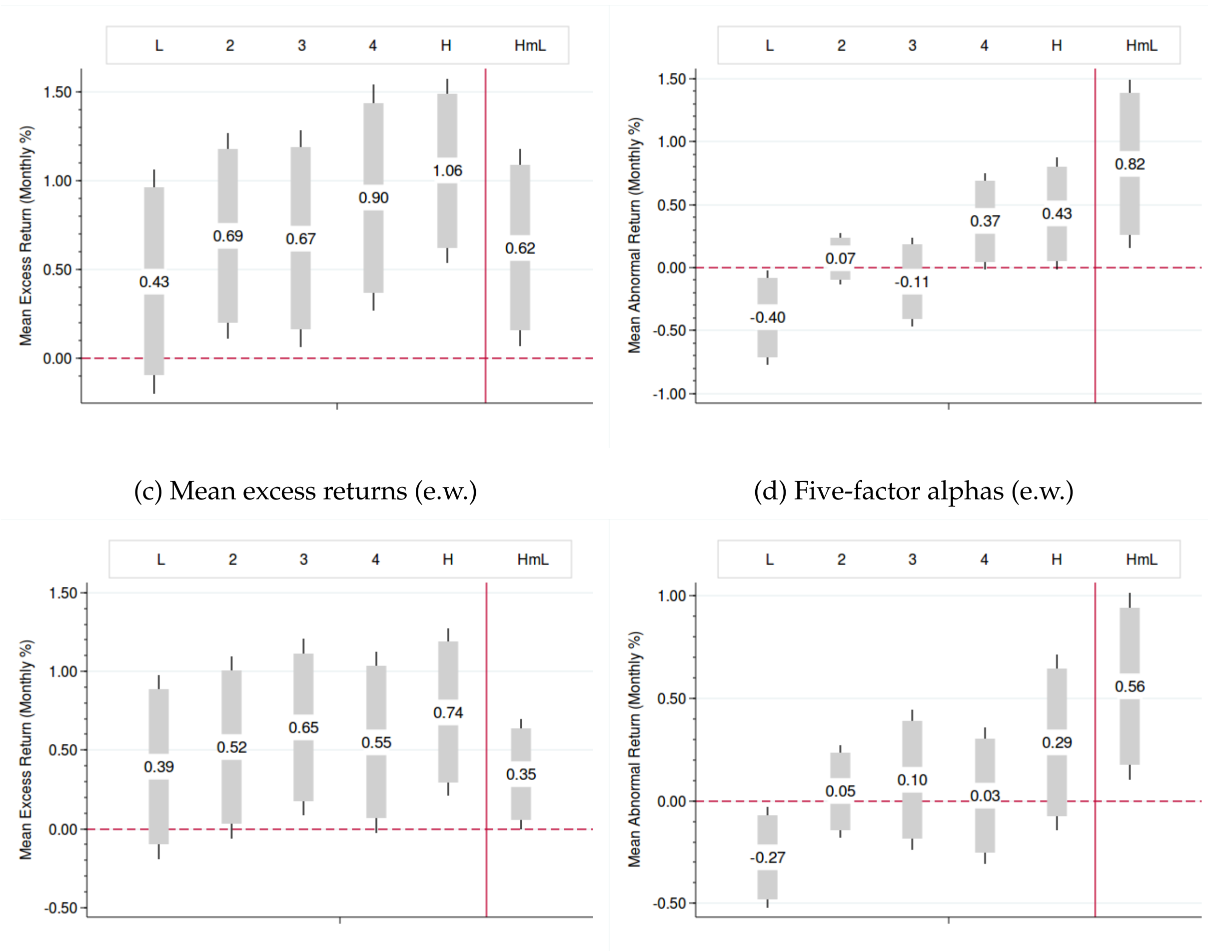


Figure 7: **Government dependency portfolios: average returns and alphas.** Panel (a) and (c) display the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Panel (b) and (d) display the alphas estimated from fitting a five-factor model to these portfolio returns; the five risk factors are the market, size, and value factors from Fama and French (1993); the momentum factor from Carhart (1997); and the liquidity factor from Pastor and Stambaugh (2003). Also displayed are 90% and 95% confidence intervals (indicated by the grey bar and the whiskers, respectively) computed with heteroskedasticity- and autocorrelation-consistent (HAC) standard errors following the routine of Newey and West (1987, 1994). Returns are monthly. Portfolios are value-weighted in panel (a) and (b), and equal-weighted in panel (c) and (d). The first portfolio formation was at the end of June in 1981, and it was based on government dependency ($\overline{StG}_{-2,0}$) computed for 1980; the same procedure are repeated every year thereafter until 2018.

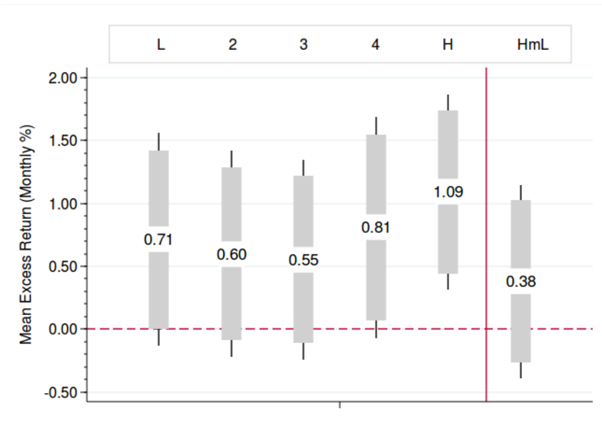
Table 6: **Government dependency portfolios: value-weighted portfolios.** Panel (a) reports the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Also reported are Sharpe ratios calculated from monthly returns but expressed in annualized percentages, and β_{Pub} obtained from time-series regressions of portfolio returns on $PubFac$, $UncFac$, and the market excess return. Panel (b) reports the estimation results of regressing these portfolio returns on five classic risk factors including the market, size, and value factors (MKT, SMB, HML) from Fama and French (1993); the momentum factor (MOM) from Carhart (1997); and the liquidity factor (LIQ) from Pastor and Stambaugh (2003). In square brackets are t-statistics computed with heteroskedasticity- and autocorrelation-consistent (HAC) standard errors following the routine of Newey and West (1987, 1994). Returns are monthly. Portfolios are value-weighted. Risk factor data are obtained from Kenneth French's and Lubos Pastor's websites. The first portfolio formation was at the end of June in 1981, and it was based on government dependency ($\overline{StG}_{-2,0}$) computed for 1980; the same procedure are repeated every year thereafter until 2018.

	Govt. dependency portfolios					High *minus* Low
	1 (low)	2	3	4	5 (high)	
(a) Return moments						
Mean excess return *(monthly %)*	0.43	0.69	0.67	0.90	1.06	0.62
Sharpe ratio *(annualized %)*	22.01	38.11	35.57	45.55	65.50	36.14
β_{Pub}	-1.23	-0.42	-0.53	0.38	1.96	3.19
(b) Controlling for classic risk factors						
α	-0.40	0.07	-0.11	0.37	0.43	0.82
	[-2.06]	[0.69]	[-0.63]	[1.89]	[1.89]	[2.42]
β_{MKT}	1.19	1.11	1.18	0.95	0.89	-0.30
	[23.59]	[22.09]	[19.88]	[17.48]	[15.96]	[-3.93]
β_{SMB}	0.29	0.18	0.15	0.45	0.18	-0.10
	[3.65]	[2.01]	[1.72]	[2.54]	[2.74]	[-0.92]
β_{HML}	-0.05	0.01	0.18	-0.22	0.23	0.28
	[-0.62]	[0.08]	[1.49]	[-1.33]	[1.49]	[1.60]
β_{MOM}	-0.03	-0.18	-0.13	-0.07	0.07	0.10
	[-0.51]	[-2.84]	[-2.23]	[-1.63]	[0.70]	[0.90]
β_{LIQ}	0.20	0.01	0.12	0.02	-0.12	-0.32
	[2.35]	[0.19]	[1.49]	[0.26]	[-1.36]	[-2.73]
Adj. R^2	0.67	0.68	0.66	0.51	0.47	0.13

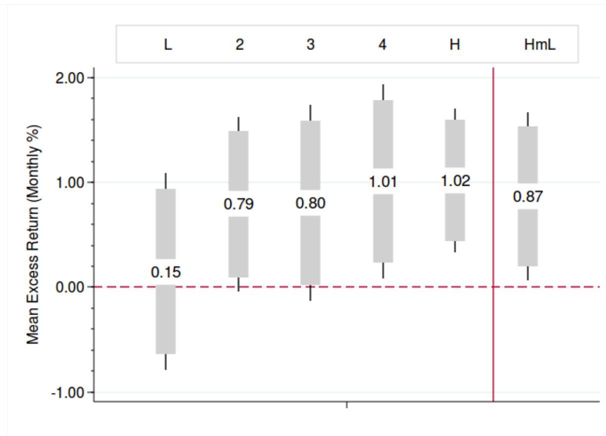
Table 7: **Government dependency portfolios: equal-weighted portfolios.** Panel (a) reports the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Also reported are Sharpe ratios calculated from monthly returns but expressed in annualized percentages. Panel (b) reports the estimation results of regressing these portfolio returns on five classic risk factors. Portfolios are equal-weighted. Other specifics are the same as in Table 6.

	Govt. dependency portfolios					High *minus* Low
	1 (low)	2	3	4	5 (high)	
(a) Return moments						
Mean excess return *(monthly %)*	0.41	0.53	0.66	0.57	0.76	0.35
Sharpe ratio *(annualized %)*	22.49	29.44	37.72	32.02	45.45	31.70
β_{Pub}	1.86	1.74	1.63	3.30	3.46	1.60
(b) Controlling for classic risk factors						
α	-0.26	0.06	0.12	0.05	0.30	0.56
	[-2.04]	[0.52]	[0.67]	[0.29]	[1.40]	[2.40]
β_{MKT}	1.03	0.93	0.95	0.86	0.81	-0.22
	[32.63]	[24.28]	[30.79]	[19.31]	[15.98]	[-3.90]
β_{SMB}	0.81	0.88	0.83	1.02	0.76	-0.06
	[12.85]	[11.20]	[10.52]	[20.75]	[8.11]	[-0.89]
β_{HML}	0.03	-0.04	-0.02	-0.05	-0.04	-0.07
	[0.39]	[-0.46]	[-0.28]	[-0.62]	[-0.42]	[-1.10]
β_{MOM}	-0.14	-0.26	-0.17	-0.16	-0.09	0.05
	[-3.09]	[-6.87]	[-2.98]	[-5.82]	[-1.41]	[1.16]
β_{LIQ}	0.09	-0.03	-0.02	0.05	-0.09	-0.18
	[1.30]	[-0.64]	[-0.44]	[0.88]	[-1.81]	[-3.40]
Adj. R^2	0.80	0.79	0.80	0.78	0.67	0.10

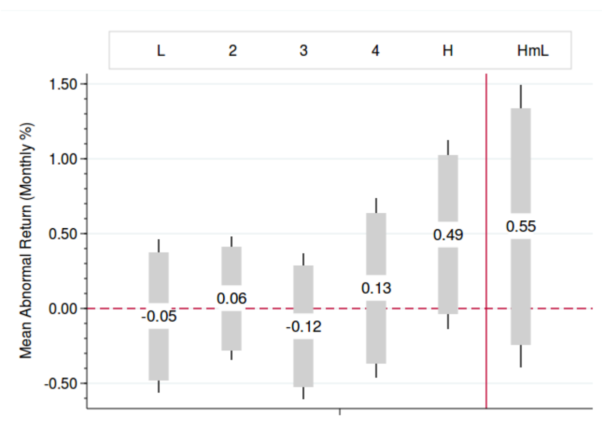
(a) Mean excess returns (1981-1999)



(b) Mean excess returns (2000-2018)



(c) Five-factor alphas (1981-1999)



(d) Five-factor alphas (2000-2018)

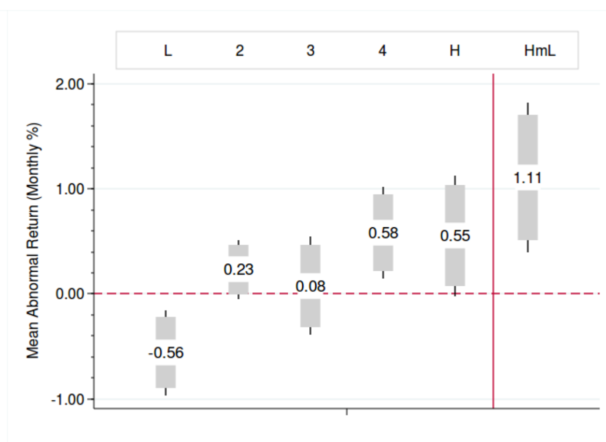


Figure 8: **Government dependency portfolios: value-weighted portfolios; subperiods: 1981 to 1999 vs. 2000 to 2018.** Panel (a) and (b) display the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Panel (c) and (d) display the alphas estimated from fitting a five-factor model to these portfolio returns. Also displayed are 90% and 95% confidence intervals indicated by the grey bar and the whiskers, respectively. The sample period is 1981 to 1999 in panel (a) and (c), and 2000 to 2018 in panel (b) and (d). Portfolios are value-weighted. Other specifics are the same as in Figure 7.

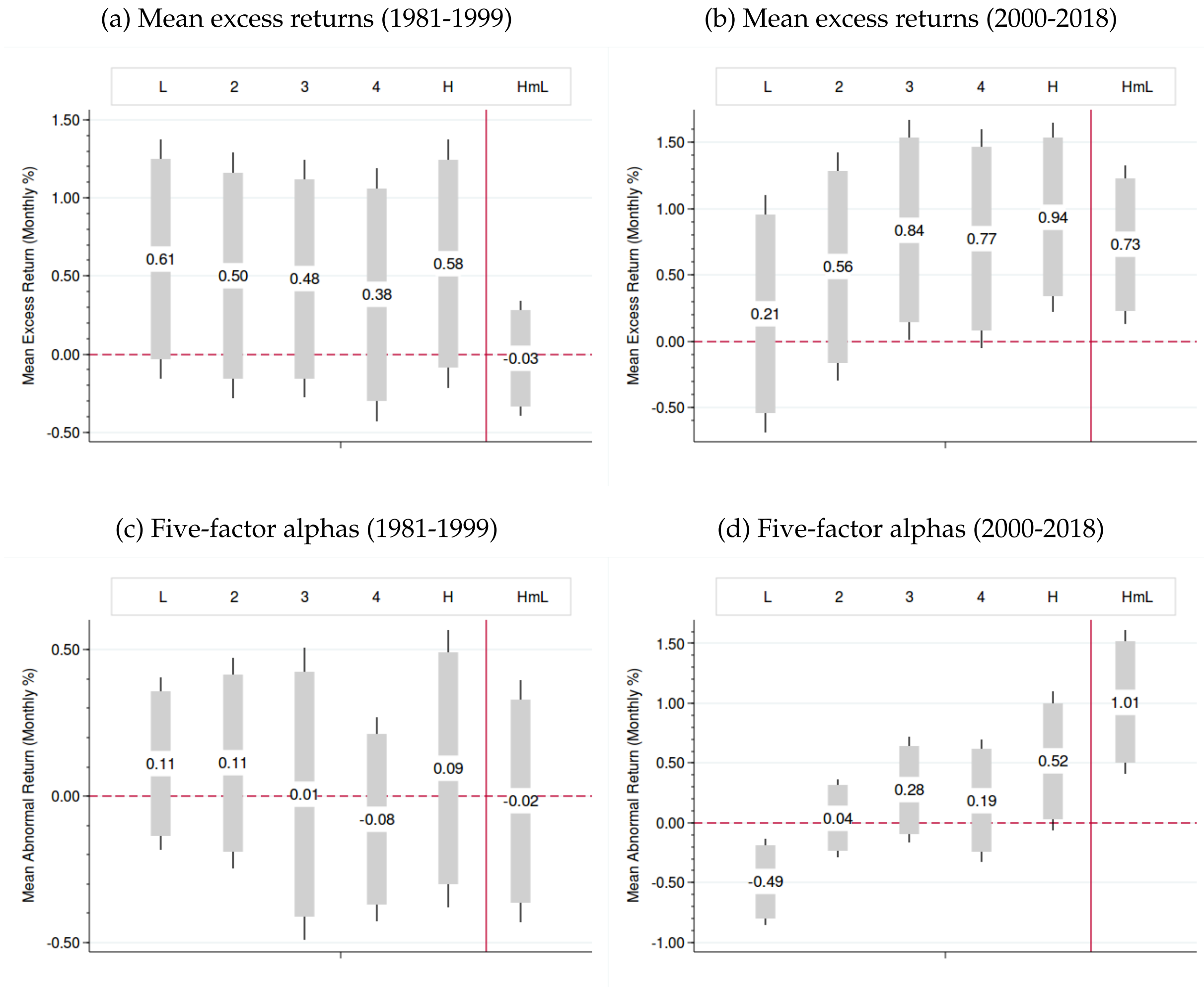


Figure 9: **Government dependency portfolios: equal-weighted portfolios; subperiods: 1981 to 1999 vs. 2000 to 2018.** Panel (a) and (b) display the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Panel (c) and (d) display the alphas estimated from fitting a five-factor model to these portfolio returns. Also displayed are 90% and 95% confidence intervals indicated by the grey bar and the whiskers, respectively. The sample period is 1981 to 1999 in panel (a) and (c), and 2000 to 2018 in panel (b) and (d). Portfolios are equal-weighted. Other specifics are the same as in Figure 7.

Table 8: **Government dependency portfolios: 1981-1999 vs. 2000-2018.** Panel (a) reports for two subperiods, 1981-1999 and 2000-2018, the mean excess returns on government dependency portfolios as well as the mean return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. Panel (b) reports the corresponding alphas estimated by regressing these portfolio returns on five classic risk factors. Portfolios are either value-weighted or equal-weighted. Other specifics are the same as in Table 6.

	Govt. dependency portfolios					High *minus* Low
	1 (low)	2	3	4	5 (high)	
(a) Mean excess return (monthly %)						
Value weight						
1981-1999	0.71	0.60	0.55	0.81	1.09	0.38
2000-2018	0.15	0.79	0.80	1.01	1.02	0.87
Equal weight						
1981-1999	0.61	0.50	0.48	0.38	0.58	-0.03
2000-2018	0.21	0.56	0.84	0.77	0.94	0.73
(b) Alphas w.r.t. five classic risk factors						
α (v.w., 1981-1999)	-0.05	0.06	-0.12	0.13	0.49	0.55
	[-0.21]	[0.31]	[-0.49]	[0.44]	[1.53]	[1.14]
α (v.w., 2000-2018)	-0.56	0.23	0.08	0.58	0.55	1.11
	[-2.71]	[1.64]	[0.33]	[2.64]	[1.90]	[3.08]
α (e.w., 1981-1999)	0.11	0.11	0.01	-0.08	0.09	-0.02
	[0.74]	[0.62]	[0.03]	[-0.44]	[0.39]	[-0.08]
α (e.w., 2000-2018)	-0.49	0.04	0.28	0.19	0.52	1.01
	[-2.68]	[0.23]	[1.24]	[0.73]	[1.76]	[3.30]

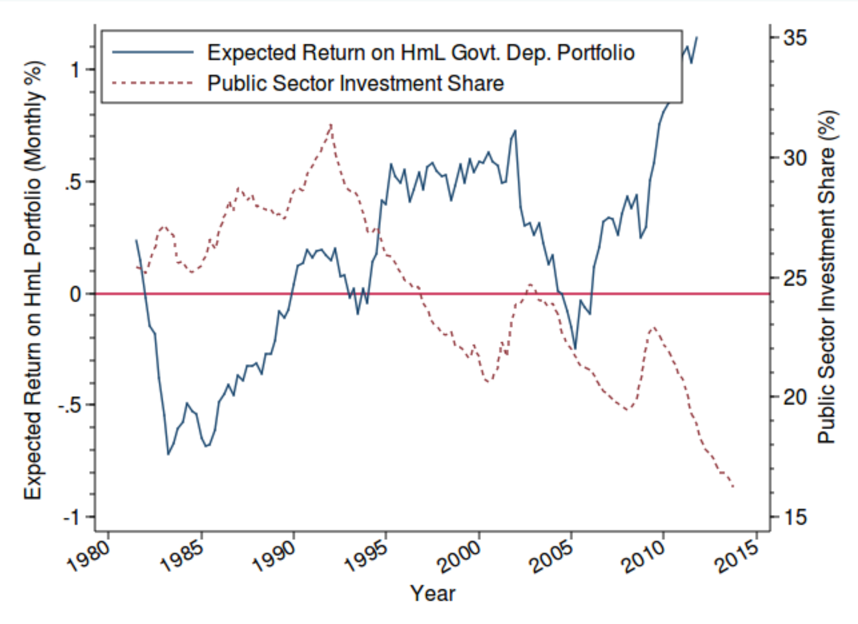


Figure 10: **Expected return on long-short government dependency portfolio and the public sector investment share.** The solid line represents the average future return (over the subsequent seven years) on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile. The dashed line represents the public sector investment share, that is, the ratio of public sector investment to the sum of public and private sector investments. The magnitude of the former (in monthly percent) is indicated on the left axis while the latter (in percent) on the right axis.

5 Conclusion

In this paper, I assess the overall (in)adequacy of public sector capital through the lens of asset prices. I develop a parsimonious two-sector GE model that links the supply of public sector capital to investors' utility. In particular, I demonstrate how investors may view the risk to public investment differently when public sector capital is under- or over-supplied, and how their views may be reflected in asset prices. Backed by this GE theory I propose a factor pricing model and confront it with a wide range of test assets. The results indicate that shocks to the public sector investment share are priced in the cross-section of stock returns with a consistently *positive* price of risk. This finding points to *increases* in public investment as good news for investors. To strengthen and expand this finding, I conduct a portfolio analysis using a sample of U.S. government contractors. I find that firms with heavier reliance on the U.S. government for revenue are more sensitive to changes in public investment and provide higher stock returns on average. I also find that the spread in average returns on high- and low-government-dependency stocks has widened in recent years, implying a bigger shortfall in public sector capital.

That said, one should not use my findings to guide the investment decision on a particular public sector project, which ought to be based on specific cost-benefit analyses. My results should instead be interpreted as an indicator of an overall undersupply of public sector capital, and that expanding public investment may generate a net benefit.

An unanswered question in this study is why the public sector is underinvested. In theory, an inadequate supply of public sector capital should attract more investment for its high marginal product (as well as other benefits). But even though the public sector investment share has been declining since the 1960s, the public investment growth remains pretty steady with no sign of a pickup whatsoever (see B.4). What is missing here? I can think of two possible drivers. One is political factors. Public investment decision-making is often influenced by political considerations that dominate economic ones in many cases, if not all. For one thing, when it comes to winning votes, tax cuts are arguably more appealing than infrastructure spending. Another reason is that inefficiencies and perversities attending the existing public sector projects may stymie any attempt to increase spending. One can reasonably argue that resolving these problems should take priority over passing big spending bills. In any case, the evidence provided in this paper suggests that augmenting public sector capital, by either investing more or spending more efficiently, has a nontrivial, positive impact on investors' welfare.

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A Appendix

In this appendix, I present expressions omitted in the main text. I also provide details on the calibration of the two-sector general equilibrium model.

A.1 Omitted expressions

The first set of omitted expressions are the drift and diffusion coefficients of ξ_t and c_t.

$$\mu_{\xi,t} = \frac{\partial_\chi \xi_t}{\xi_t}\mu_{\chi,t} + \frac{\partial_\sigma \xi_t}{\xi_t}\kappa(\bar{\sigma} - \sigma_t) + \frac{1}{2}\frac{\partial_{\chi\chi}\xi_t}{\xi_t}\sigma_{\chi,t}^2 + \frac{1}{2}\frac{\partial_{\sigma\sigma}\xi_t}{\xi_t}\nu^2\sigma_t$$

$$\sigma_{1,t}^{\xi} = \frac{\partial_\sigma \xi_t}{\xi_t}\nu\sqrt{\sigma_t} \qquad \sigma_{2,t}^{\xi} = \frac{\partial_\chi \xi_t}{\xi_t}\sigma_{\chi,t}$$

$$\mu_{c,t} = \frac{\partial_\chi c_t}{c_t}\mu_{\chi,t} + \frac{\partial_\sigma c_t}{c_t}\kappa(\bar{\sigma} - \sigma_t) + \frac{1}{2}\frac{\partial_{\chi\chi}c_t}{c_t}\sigma_{\chi,t}^2 + \frac{1}{2}\frac{\partial_{\sigma\sigma}c_t}{c_t}\nu^2\sigma_t$$

$$\sigma_{1,t}^{c} = \frac{\partial_\sigma c_t}{c_t}\nu\sqrt{\sigma_t} \qquad \sigma_{2,t}^{c} = \frac{\partial_\chi c_t}{c_t}\sigma_{\chi,t}$$

Next is the pricing kernel Λ_t, which is defined as per Duffie and Epstein (1992)

$$\Lambda_t = \exp\left[\int_0^t u_V(C_\tau, V_\tau)d\tau\right]u_C(C_t, V_t)$$

with

$$u_C(C,V) \equiv \frac{\partial u(C,V)}{\partial C} = \frac{\beta C^{-1/\psi}}{[(1-\gamma)V]^{\frac{\gamma-1/\psi}{1-\gamma}}}$$

$$u_V(C,V) \equiv \frac{\partial u(C,V)}{\partial V} = \frac{\beta}{1-1/\psi}\left[(1/\psi - \gamma)\frac{C^{1-1/\psi}}{[(1-\gamma)V]^{\frac{1-1/\psi}{1-\gamma}}} - (1-\gamma)\right].$$

The law of motion of the pricing kernel can be derived using Ito's lemma

$$\frac{d\Lambda_t}{\Lambda_t} = u_V(C_t, V_t)dt + \frac{du_C(C_t, V_t)}{u_C(C_t, V_t)} = -r_t dt - \eta_t^K dZ_t - \eta_t^\sigma dZ_t^\sigma - \eta_t^\chi dW_t.$$

One can easily verify that when $1/\psi = \gamma$, these three expressions collapse to those derived from a standard continuous-time Lucas (1978) economy with power utility.

A.2 Model calibration

Table A.1: **Baseline calibration**

Parameter	Variable	Value
Capital accumulation:		
Capital depreciation rate	δ	0.05
Capital adjustment costs*	ϱ	1.07
Size of PUB shocks	ς	0.05
Uncertainty dynamics:		
Mean-reversion parameter	κ	0.16
Long-run mean of uncertainty*	$\bar{\sigma}$	0.02
Volatility parameter*	ν	0.07
Correlation with aggregate shocks*	$\rho_{K\sigma}$	-0.23
Preferences:		
Subjective time discount	β	0.01
Relative risk aversion	γ	9
Elasticity of intertemporal substitution (EIS)	ψ	2
Aggregate production:		
Scale parameter*	m	0.215
Share parameter*	α	0.34
Substitutability parameter*	s	3.2

B Online Appendix

In this online appendix, I provide a heuristic derivation of the HJB equation associated with the utility maximization problem of an agent with recursive preferences. I also empirically examine the relationships between the public sector investment share, the real risk-free rate, and economic uncertainty, which turn out to be consistent with the model predictions. In addition, I provide details on the numerical solution of the two-sector GE model. Lastly, I elaborate on the construction of the government contractor sample and the calculations of related variables. Additional empirical results, tables and figures, are also presented here.

B.1 Derivation of the HJB Equation with Recursive Preferences

I start from a discrete-time setting and derive the continuous-time limit, following a similar route as Obstfeld (1994); technical details are addressed by Duffie and Epstein (1992).

Consider the utility maximization problem of an agent with recursive preferences:

$$\mathcal{V}_t = \max \left[(1 - e^{-\beta\Delta}) C_t^{1-1/\psi} + e^{-\beta\Delta} (\mathbb{E}_t \mathcal{V}_{t+\Delta}^{1-\gamma})^{\frac{1-1/\psi}{1-\gamma}} \right]^{\frac{1}{1-1/\psi}} \tag{B.1}$$

where the time length per period is Δ, and other parameters are defined as usual. Define a new value function $V_t \equiv \frac{\mathcal{V}_t^{1-\gamma}}{1-\gamma}$, and rewrite (B.1) as

$$[(1-\gamma)V_t]^{\frac{1}{1-\gamma}} = \max\left\{(1-e^{-\beta\Delta})C_t^{1-1/\psi} + e^{-\beta\Delta}[\mathbb{E}_t(1-\gamma)V_{t+\Delta}]^{\frac{1-1/\psi}{1-\gamma}}\right\}^{\frac{1}{1-1/\psi}}.$$

Define another function $G(X) \equiv [(1-\gamma)X]^{\frac{1-1/\psi}{1-\gamma}}$, and rewrite again:

$$G(V_t)^{\frac{1}{1-1/\psi}} = \max\left[(1-e^{-\beta\Delta})C_t^{1-1/\psi} + e^{-\beta\Delta}G(\mathbb{E}_t V_{t+\Delta})\right]^{\frac{1}{1-1/\psi}}. \tag{B.2}$$

Because $\frac{X^{1-1/\psi}}{1-1/\psi}$ is a monotonic transformation of X, maximizing $G(V_t)^{\frac{1}{1-1/\psi}}$ and $\frac{G(V_t)}{1-1/\psi}$ are equivalent; so (B.2) is equivalent to

$$\frac{G(V_t)}{1-1/\psi} = \max\left[(1-e^{-\beta\Delta})\frac{C_t^{1-1/\psi}}{1-1/\psi} + e^{-\beta\Delta}\frac{G(\mathbb{E}_t V_{t+\Delta})}{1-1/\psi}\right]. \tag{B.3}$$

Subtract $e^{-\beta\Delta}\frac{G(V_t)}{1-1/\psi}$ from both sides:

$$(1-e^{-\beta\Delta})\frac{G(V_t)}{1-1/\psi} = \max\left\{(1-e^{-\beta\Delta})\frac{C_t^{1-1/\psi}}{1-1/\psi} + e^{-\beta\Delta}\left[\frac{G(\mathbb{E}_t V_{t+\Delta})}{1-1/\psi} - \frac{G(V_t)}{1-1/\psi}\right]\right\}.$$

Divide both sides by Δ and take $\Delta \to 0$:

$$\lim_{\Delta\to 0}\frac{1-e^{-\beta\Delta}}{\Delta}\frac{G(V_t)}{1-1/\psi} = \max\left\{\lim_{\Delta\to 0}\frac{1-e^{-\beta\Delta}}{\Delta}\frac{C_t^{1-1/\psi}}{1-1/\psi} + \lim_{\Delta\to 0}e^{-\beta\Delta}\frac{\frac{G(\mathbb{E}_t V_{t+\Delta})}{1-1/\psi} - \frac{G(V_t)}{1-1/\psi}}{\Delta}\right\}.$$

Use Taylor's theorem:

$$\frac{\beta}{1-1/\psi}G(V_t) = \max\left\{\frac{\beta}{1-1/\psi}C_t^{1-1/\psi} + \frac{1}{1-1/\psi}G'(V_t)\frac{\mathbb{E}_t dV_t}{dt}\right\}.$$

Substitute in function $G(\cdot)$ and rearrange terms:

$$0 = \max\left\{\frac{\beta(1-\gamma)V_t}{1-1/\psi}\left\{\frac{C_t^{1-1/\psi}}{[(1-\gamma)V_t]^{\frac{1-1/\psi}{1-\gamma}}} - 1\right\} + \frac{\mathbb{E}_t dV_t}{dt}\right\}. \tag{B.4}$$

From (B.4), I obtain equation (11) in the main text.

B.2 Testing model predictions

The GE model presented in the paper predicts that, when facing greater uncertainty, the public sector investment share rises while the risk-free rate declines; but controlling for uncertainty, it predicts a positive association between these two variables (see Figure **??**). Here I take this prediction to the U.S. data.

B.2.1 Specifications

I start by examining the role of uncertainty as a predictor of the public sector investment share and the real risk-free rate. I use a standard predictive regression specified as

$$\mathcal{A}^h(Y_t) = \alpha + \beta \times UNC_t + \epsilon_{t+h}, \tag{B.5}$$

where $\mathcal{A}^h(Y_t) \equiv \frac{1}{h+1}\sum_{\tau=0}^{h} Y_{t+\tau}$ is the average value of a predicted variable Y over a forecast horizon of h periods (e.g., $\mathcal{A}^1(Y_t) = (Y_t + Y_{t+1})/2$), UNC_t is an uncertainty index from Jurado, Ludvigson, and Ng (2015), and ϵ_{t+h} is the forecast error. The predicted variables include $PubIS_t^{cyc}$, the cyclical component of the public sector investment share, and r_t, the real risk-free rate. All variables are already defined in Section 3 and Appendix A.

I then test the relation between the public sector investment share and the real risk-free rate controlling for uncertainty. Specifically, I estimate the following regression:

$$\mathcal{A}^h(r_t) = \alpha + \beta_1 \times \mathcal{A}^h(PubIS_t^{cyc}) + \beta_2 \times UNC_t + \epsilon_t. \tag{B.6}$$

My main interest is the slope coefficient β_1, which is predicted to be positive according to my model. I run this regression under different horizons because, in practice, both the public sector investment share and the risk-free rate may not respond instantaneously to changes in economic conditions. Allowing some flexibility in the time frame may help identify the correlation implied by the model.

B.2.2 Results

Table B.1 presents the estimation results based on a sample from 1960Q3 to 2018Q4; the first observation is dictated by the start of the uncertainty measure. I trimmed the 1979Q4 to 1982Q4 episode to avoid a spell of drastic movements in interest rates caused by a well-documented monetary policy shock. [31] My model does not incorporate monetary policy

[31] Clarida, Gali, and Gertler (2000) point out that this episode was characterized by a sharp, one-shot "Volcker shock" that brought inflation down by more than 5 percent in a relatively short period of time. Also, the operating procedures of the Federal Reserve briefly changed to targeting non-borrowed reserves

risk, so it cannot speak to changes in that period.

Conforming to the model prediction, panel (a) in Table B.1 shows that the public sector investment share and the real risk-free rate react differently to higher uncertainty: the former goes up, whereas the latter goes down. The estimated slope coefficients are statistically significant for all horizons, and their magnitudes increase in horizon. As for the economic significance, at the two-year horizon, a one-standard-deviation (≈ 0.075) increase in the JLN uncertainty index is associated with a 66 basis point (bps) decrease in the (annualized) real risk-free rate and a 0.67 percentage point increase in the public sector investment share. The adjusted R^2 also increases in horizon, ranging from 0.08 to 0.14 for the real risk-free rate and 0.08 to 0.17 for the public sector investment share.

Panel (b) examines the relation between the real risk-free rate and the public sector investment share. As shown, without any control, the real risk-free rate is barely related to the contemporaneous public sector investment share for all horizons. But controlling for uncertainty, the real risk-free rate displays a positive association with the public sector investment share. In particular, at the two-year horizon, a one-standard-deviation ($\approx$ 2.2%) increase in the public sector investment share is associated with a 88 bps higher real risk-free rate. This is again consistent with the model prediction.

B.3 Numerical Methods

The two-sector general equilibrium model presented in the paper is numerically solved using an iterative method. The procedure is as follows. I start by putting together a system of partial differential equations (PDEs) that characterizes a Markov equilibrium. It consists of the HJB equation associated with the central planning problem and the corresponding first-order conditions (FOCs):

$$\begin{aligned}\frac{\beta}{1-1/\psi} = \max_{\iota_t^p,\,\iota_t^g} \frac{\beta}{1-1/\psi}\left(\frac{c_t}{\xi_t}\right)^{1-1/\psi} + \mu_{K,t} + \mu_{\xi,t} - \frac{\gamma}{2}[\sigma^2 + (1-\chi_t)^2\varsigma_t^2 + \sigma_{\xi,t}^2 + \varsigma_{\xi,t}^2] \\ + (1-\gamma)(\sigma\sigma_{\xi,t} + (1-\chi_t)\varsigma_t\varsigma_{\xi,t}),\end{aligned} \tag{B.7}$$

$$\left(\frac{c_t}{\xi_t}\right)^{1/\psi} = \frac{\beta}{\phi'(\iota_t^p)}\frac{1}{\xi_t - \chi_t\,\partial_\chi\xi_t} \qquad \left(\frac{c_t}{\xi_t}\right)^{1/\psi} = \frac{\beta}{\phi'(\iota_t^g)}\frac{1}{\xi_t + (1-\chi_t)\partial_\chi\xi_t} \tag{B.8}$$

in lieu of the usual instrument, Federal Funds rate. These monetary factors caused exceptional disturbances to the real interest rates. Also see Christiano, Eichenbaum, and Evans (1999) and Romer (2016).

Table B.1: **Interest rate, public sector investment share, and economic uncertainty.** Panel (a) reports the estimation results of a predictive regression (B.5). The dependent variable is $\mathcal{A}^h(Y_t)$, the average value of a predicted variable Y over a forecast horizon of h periods; Y is either the (annualized) real risk-free rate or the cyclical component of the public sector investment share, and h equals 2, 4, or 8 quarters. The regressor (UNC) is an economic uncertainty index from Jurado, Ludvigson, and Ng (2015). Panel (b) reports the estimation results of another regression (B.6). The t-statistics are based on heteroskedasticity- and autocorrelation-consistent (HAC) standard errors (Newey and West, 1987, 1994). The sample is from 1960Q3 to 2018Q4 with the period from 1979Q4 to 1982Q4 trimmed due to a significant monetary policy shock.

(a) Economic uncertainty as a predictor

	Real risk-free rate (annualized, %)		
Forecast horizon (h)	2-quarter	4-quarter	8-quarter
UNC	-7.46	-7.97	-8.76
$[t]$	[-2.23]	[-2.32]	[-3.00]
Adj. R^2	0.08	0.10	0.14

	Public sector inv. share (cyc., %)		
Forecast horizon (h)	2-quarter	4-quarter	8-quarter
UNC	7.89	9.89	9.03
$[t]$	[3.20]	[4.96]	[4.98]
Adj. R^2	0.08	0.15	0.18

(b) The relation between the risk-free rate and the public sector investment share

	Real risk-free rate (annualized, %)					
Forecast horizon (h)	2-quarter		4-quarter		8-quarter	
$\mathcal{A}^h(PubIS^{cyc})$	0.09	0.19	0.09	0.26	0.15	0.40
$[t]$	[0.69]	[1.41]	[0.64]	[1.81]	[0.82]	[2.59]
UNC		-8.93		-10.48		-12.36
$[t]$		[-3.37]		[-3.62]		[-4.22]
Adj. R^2	0.01	0.11	0.01	0.15	0.01	0.25

Interpretation: Greater uncertainty precedes a lower risk-free rate but a higher public sector investment share. Controlling for uncertainty, a higher public sector investment share coincides with a higher risk-free rate.

where $c_t \equiv [M(\chi_t) - \iota_t^p \chi_t - \iota_t^g (1-\chi_t)]$ is the consumption-capital ratio, and $\xi_t \equiv \xi(\chi_t, \varsigma_t)$ is the unknown function to be obtained. Ideally, with the state of this system determined by χ_t and ς_t, one should seek the true solution—that is, a well-behaved analytical function $\xi^*(\chi_t, \varsigma_t)$ that satisfies (B.7) and (B.8). But in this case such a solution is difficult to find, if not impossible. Thus my goal instead is to find a numerical solution that approximates the true solution as close as possible.

Discretization. The first step is to choose a set of grid points in the state space. Specifically, I choose $\mathcal{I} \times \mathcal{J}$ grid points from the state space; each point, denoted by (i, j), represents a unique state of the system characterized by $\chi(i)$ and $\varsigma(j)$, where

$$\chi(i) = 3\frac{i^2}{\mathcal{I}^2} - 2\frac{i^3}{\mathcal{I}^3}, \quad i = 1, ..., \mathcal{I}, \qquad\qquad \varsigma(j) = \frac{j^2}{\mathcal{J}^2}, \quad j = 1, ..., \mathcal{J}.$$

This scheme construct a nonuniform grid that is denser near boundaries where function ξ is expected to have more curvature. [32] Alternatively, one can also use uniform grids that are simpler to construct but lend less accuracy.

Iterative method. The next step is to find the approximate values of function ξ at these grid points. I adapt an iterative method from Brunnermeier and Sannikov (2016a) and Achdou et al. (2017); the key idea is to add a pseudo time dimension to the system and iterate it until convergence. Specifically, I assume that ξ is directly dependent on time, that is, ξ_t equals $\xi(\chi_t, \varsigma_t, t)$ instead of $\xi(\chi_t, \varsigma_t)$. Then I modify equation (B.7) accordingly and write it as a linear combination of the first- and second-order partial derivatives of ξ:

$$H_{0,t} = \frac{\partial \xi_t}{\partial t} + H_{1,t}\frac{\partial \xi_t}{\partial \chi_t} + H_{2,t}\frac{\partial \xi_t}{\partial \varsigma_t} + H_{3,t}\frac{\partial^2 \xi_t}{\partial \chi_t^2} + H_{4,t}\frac{\partial^2 \xi_t}{\partial \varsigma_t^2} \tag{B.9}$$

[32] See Brunnermeier and Sannikov (2016b) for another example of using this scheme. There is a whole area of research concentrated on the optimal grid generation (see, e.g., Thompson, Warsi, and Mastin, 1985). The presented method may not be optimal but works well enough in this context.

where

$$
\begin{aligned}
H_{0,t} =& \xi_t \Bigg\{ \frac{\beta}{1-1/\psi} - \frac{\beta}{1-1/\psi}\Big(\frac{c_t}{\xi_t}\Big)^{1-1/\psi} - [(1-\chi_t)\phi(\iota_t^p) + \chi_t\phi(\iota_t^g) - \delta] \\
& + \frac{\gamma}{2}\Big[\sigma^2 + (1-\chi_t)^2\varsigma_t^2 + \Big(\frac{\partial_\varsigma \xi_t}{\xi_t}\Big)^2 \nu^2\varsigma_t + \Big(\frac{\partial_\chi \xi_t}{\xi_t}\Big)^2 \varsigma_{\chi,t}^2\Big]\Bigg\} \\
H_{1,t} =& \mu_{\chi,t} - (1-\gamma)(1-\chi_t)\varsigma_t\varsigma_{\chi,t} \\
H_{2,t} =& \kappa(\bar{\varsigma} - \varsigma_t) - (1-\gamma)\sigma\nu\sqrt{\varsigma_t} \\
H_{3,t} =& \frac{\varsigma_{\chi,t}^2}{2} \\
H_{4,t} =& \frac{\nu^2\varsigma_t}{2}.
\end{aligned}
\tag{B.10}
$$

The core step is to design an algorithm that takes in some guessed values of ξ and generates updated ones, for which there are two options: the explicit and implicit methods.

The explicit method is relatively easy to implement. Specifically, I evaluate the revised HJB equation (B.9) at every grid point, transforming it into a set of difference equations. For a given grid point (i, j), I substitute $\chi(i)$, $\varsigma(j)$, and the guessed value of $\xi(i, j)$ into (B.8), (B.9), and (B.10) to attain a difference equation:

$$
H_0(i,j) = \frac{\partial \xi}{\partial t}\bigg|_{(i,j)} + H_1(i,j)\frac{\partial \xi}{\partial \chi}\bigg|_{(i,j)} + H_2(i,j)\frac{\partial \xi}{\partial \varsigma}\bigg|_{(i,j)} + H_3(i,j)\frac{\partial^2 \xi}{\partial \chi^2}\bigg|_{(i,j)} + H_4(i,j)\frac{\partial^2 \xi}{\partial \varsigma^2}\bigg|_{(i,j)},
\tag{B.11}
$$

where the derivatives are approximated using the finite difference method. [33]

$$
\frac{\partial \xi}{\partial \chi}\bigg|_{(i,j)} \approx \begin{cases} \frac{\xi(i+1,j)-\xi(i,j)}{\chi(i+1)-\chi(i)}, & i=1 \\ \frac{\xi(i+1,j)-\xi(i-1,j)}{\chi(i+1)-\chi(i-1)}, & 1<i<\mathcal{I} \\ \frac{\xi(i,j)-\xi(i-1,j)}{\chi(i)-\chi(i-1)}, & i=\mathcal{I} \end{cases}
\qquad
\frac{\partial \xi}{\partial \varsigma}\bigg|_{(i,j)} \approx \begin{cases} \frac{\xi(i,j+1)-\xi(i,j)}{\varsigma(j+1)-\varsigma(j)}, & j=1 \\ \frac{\xi(i,j+1)-\xi(i,j-1)}{\varsigma(j+1)-\varsigma(j-1)}, & 1<j<\mathcal{J} \\ \frac{\xi(i,j)-\xi(i,j-1)}{\varsigma(j)-\varsigma(j-1)}, & j=\mathcal{J} \end{cases}
$$

$$
\frac{\partial^2 \xi}{\partial \chi^2}\bigg|_{(i,j)} \approx \begin{cases} \frac{[\chi(i+1)-\chi(i)]\xi(i+2,j)-[\chi(i+2)-\chi(i)]\xi(i+1,j)+[\chi(i+2)-\chi(i+1)]\xi(i,j)}{\frac{1}{2}[\chi(i+2)-\chi(i)][\chi(i+2)-\chi(i+1)][\chi(i+1)-\chi(i)]}, & i=1 \\ \frac{[\chi(i)-\chi(i-1)]\xi(i+1,j)-[\chi(i+1)-\chi(i-1)]\xi(i,j)+[\chi(i+1)-\chi(i)]\xi(i-1,j)}{\frac{1}{2}[\chi(i+1)-\chi(i-1)][\chi(i+1)-\chi(i)][\chi(i)-\chi(i-1)]}, & 1<i<\mathcal{I} \\ \frac{[\chi(i-1)-\chi(i-2)]\xi(i,j)-[\chi(i)-\chi(i-2)]\xi(i-1,j)+[\chi(i)-\chi(i-1)]\xi(i-2,j)}{\frac{1}{2}[\chi(i)-\chi(i-2)][\chi(i)-\chi(i-1)][\chi(i-1)-\chi(i-2)]}, & i=\mathcal{I} \end{cases}
$$

[33] I mainly used central differences in this paper. But I also tried the “upwind scheme”, a method that is widely considered as the most reliable one (in terms of stability) when it comes to this type of problems (Achdou et al., 2017). Since in the context of my model the central differences perform reasonably well, I skip the explanation of the “upwind scheme” for brevity.

$$\left.\frac{\partial^2 \xi}{\partial \varsigma^2}\right|_{(i,j)} \approx \begin{cases} \frac{[\varsigma(j+1)-\varsigma(j)]\xi(i,j+2)-[\varsigma(j+2)-\varsigma(j)]\xi(i,j+1)+[\varsigma(j+2)-\varsigma(j+1)]\xi(i,j)}{\frac{1}{2}[\varsigma(j+2)-\varsigma(j)][\varsigma(j+2)-\varsigma(j+1)][\varsigma(j+1)-\varsigma(j)]}, & j=1 \\ \frac{[\varsigma(j)-\varsigma(j-1)]\xi(i,j+1)-[\varsigma(j+1)-\varsigma(j-1)]\xi(i,j)+[\varsigma(j+1)-\varsigma(j)]\xi(i,j-1)}{\frac{1}{2}[\varsigma(j+1)-\varsigma(j-1)][\varsigma(j+1)-\varsigma(j)][\varsigma(j)-\varsigma(j-1)]}, & 1<j<\mathcal{J} \\ \frac{[\varsigma(j-1)-\varsigma(j-2)]\xi(i,j)-[\varsigma(j)-\varsigma(j-2)]\xi(i,j-1)+[\varsigma(j)-\varsigma(j-1)]\xi(i,j-2)}{\frac{1}{2}[\varsigma(j)-\varsigma(j-2)][\varsigma(j)-\varsigma(j-1)][\varsigma(j-1)-\varsigma(j-2)]}, & j=\mathcal{J} \end{cases}$$

$$\left.\frac{\partial \xi}{\partial t}\right|_{(i,j)} \approx \frac{\xi(i,j)-\xi^u(i,j)}{\Delta}, \qquad \Delta \text{ is the step size}^{34}$$

I first use (B.8) to attain the values of $\iota^p(i,j)$ and $\iota^g(i,j)$, which then are used to compute (B.10). Plugging (B.10) into (B.9) gives (B.11), in which the updated value—denoted by $\xi^u(i,j)$—is the only unknown and hence can be "explicitly" computed. Repeating this calculation for all grid points gives a full set of updated values, $\{\xi^u(i,j);\ i=1,...,\mathcal{I}$ and $j=1,...,\mathcal{J}\}$.

Another approach to attain updates is the implicit method. Compared with the explicit method, the only difference here is that four of the partial derivatives in (B.11) are now approximated using the updated values in lieu of the guessed ones, that is, [35]

$$H_0(i,j) = \left.\frac{\partial \xi}{\partial t}\right|_{(i,j)} + H_1(i,j)\left.\frac{\partial \xi^u}{\partial \chi}\right|_{(i,j)} + H_2(i,j)\left.\frac{\partial \xi^u}{\partial \varsigma}\right|_{(i,j)} + H_3(i,j)\left.\frac{\partial^2 \xi^u}{\partial \chi^2}\right|_{(i,j)} + H_4(i,j)\left.\frac{\partial^2 \xi^u}{\partial \varsigma^2}\right|_{(i,j)} \tag{B.12}$$

Such changes result in interdependence among the corresponding difference equations, which makes it impossible to calculate $\xi^u(i,j)$ point by point. Instead I stack all difference equations together and treat them as a system that can be written in matrix form

$$\boldsymbol{A\xi^u} = \boldsymbol{B}, \tag{B.13}$$

where $\boldsymbol{A}$ is an $(\mathcal{I}\times\mathcal{J})\times(\mathcal{I}\times\mathcal{J})$ sparse matrix, and $\boldsymbol{B}$ is an $(\mathcal{I}\times\mathcal{J})\times 1$ vector. (B.13) can be solved efficiently by taking advantage of the sparse matrix operations in Matlab. The solution $\boldsymbol{\xi^u} \equiv [\xi^u(1,1),...,\xi^u(\mathcal{I},\mathcal{J})]$ is a vector of updated values.

Summary. Put together, an algorithm to find the numerical solution to (B.7) and (B.8) is summarized below.

Start with an initial guess of ξ, follow these steps:

1. For all $i=1,...,\mathcal{I}$ and $j=1,...,\mathcal{J}$, compute ι^p and ι^g using (B.8), and H_0 to H_4 using

[34] It can be shown that the explicit method converges only if Δ is sufficiently small, while the implicit method is not subject to this constraint.

[35] Note that ξ is replaced by ξ^u at four places. Strictly speaking, the presented method is only "semi-implicit". A fully implicit method requires the partial derivatives in (B.10) to be calculated using the updated values as well. But that would produce a nonlinear optimization problem instead of the linear one presented here.

(B.10). Replace partial derivatives with finite differences.

2. Find $\xi^u(i,j)$ for every grid point using either the explicit method (B.11) or the implicit method (B.12).

3. If ξ^u is close enough to the guessed ξ, then stop. Otherwise, use ξ^u as the new guess and go back to step 1.

Several implementation notes are in order. First, although this algorithm is not rigorously validated (e.g., convergence, stability, etc.), it demonstrates smooth and stable convergence when confronted with a wide range of parameter configurations. This is especially true for the implicit method. (In comparison, the explicit method fails to converge for some parameter values.) Hence, based on my experience, the implicit method is preferred over the explicit method for its better stability as well as higher efficiency (since a larger step size can be used). But these advantages come with some cost: the implicit method is much less penetrable and harder to code and debug. So probably a better strategy is to carry out the explicit method first to help one think through the whole process. And with that as a foundation, it becomes more straightforward to modify the code and apply the implicit method.

Second, the accuracy of the numerical approximation of partial derivatives is essential to the success of this algorithm. In particular, both the implicit and explicit methods need to calculate (B.8) and (B.10) using the guessed ξ, in which the evaluations of its partial derivatives are involved. I experiment two schemes to reduce the approximation errors. The first scheme is to fit a polynomial to the guessed ξ, and then use that polynomial as a proxy to compute derivatives at any given point. The advantage of this scheme is that the derivatives are perfectly calculated with no approximation whatsoever. But it only works as well as the fitting, the performance of which drops drastically outside of the region where ξ has mild curvature. The second scheme is to apply a sophisticated interpolation method (like *spline*) to the guessed ξ, and then calculate derivatives numerically with ultra-fine grids. This scheme works reasonably well even when ξ has extreme curvature. Given the properties of these two schemes, my strategy is to start with the former (that is, when the guessed ξ is far from the exact solution) and use a small grid that only covers the region where ξ has mild curvature. Then I switch to the latter scheme, using the result from the former one as a start point and a broader grid that includes more points from uncovered region. This strategy leverages the strengths of both schemes and fares very well in my application.

B.4 Additional Details on Government Contractor Sample

This section complements my portfolio-based analysis in the main text, which uses a sample of U.S. government contractors. I provide more details on the sample construction and variable calculations.

Constructing the government contractor sample. To identify firms with sales to the U.S. government, I source accounting data from the Compustat database. I begin by selecting firms that meet standard criteria in the literature: that is, firms incorporated in the U.S. and with common stocks listed on the NYSE, AMEX, or NASDAQ; firms involved in significant mergers/acquisitions or seriously affected by the 1988 accounting change are excluded; [36] firms in the finance or utilities industry, with `SIC` $\in [6000, 7000) \cup [4900, 4950)$, are also dropped. [37] For selected firms I obtain their annual accounting records [38] from the fundamental annual file (`funda`) as well as the segment customer file (`seg_customer`); the latter provides information on firms' sales to the U.S. government (federal, state, and local). [39] These accounting data allow me to compute for each firm-year the fraction of sales accounted for by government (denoted by StG). [40] Every year I define government contractors as firms that reported positive StG at least once over the past three years; according to this definition I find about 2,400 firms. However, transactions between these firms and government may stem from various types of government expenditures that are hardly related to public sector investment. So to be more specific, I exclude firms in the healthcare and pharmaceutical industries, personal and business services industries, and the defense industry (as defined by the Fama-French 48-industry classification). I also exclude firms in the consumer goods industry (as defined by the Fama-French 5-industry classification). Government contractors in these industries are least relevant with respect to public sector investment. The resulting sample consists of 1,242 government contractors with 9,944 firm-year observations spanning 1980 to 2017.

Calculating related variables. Using the Compustat data, I calculate a selection of firm characteristics for these government contractors; the following explains the calculations in detail. StG ratio, as already mentioned, is sales to government divided by total sales

[36] If a firm experienced a *significant* merger or acquisition in a fiscal year, it would be assigned a footnote code of `AB`, `FD`, `FE`, or `FF`. According to Covas and Den Haan (2011), firms that were most affected by the 1988 accounting change (i.e., FAS94) include GM, GE, Ford, and Chrysler (also see Bernanke et al., 1990).

[37] I obtain the Standard Industrial Classification (`SIC`) code from the fundamental annual file (`funda`), or the name file (`names`) if the former is not available.

[38] I only consider records showing positive total assets (item `at`) *and* net sales (item `sale`).

[39] Data on government customers start from 1978.

[40] If no transaction with government is reported, then StG is set to zero.

(item sale). $\overline{StG}_{-2,0}$ is a 3-year trailing average of StG and serves as my measure of government dependency. The book-to-market ratio is the book value of equity divided by the market value of equity. The book value of equity is stockholders' equity (item seq) plus deferred taxes and investment tax credit (item txditc) minus preferred stock redemption/liquidation/par value (item pstkrv/pstkl/pstk). [41] The market value of equity is market price per share times number of shares outstanding; I obtain these two items from the Compustat fundamental annual file (funda), or the security monthly file (secm), or the CRSP monthly stock file (msf), based on availability in that order. The market value of equity is also referred to as market capitalization, a measure of firm size. Market leverage is the book value of debt divided by the sum of the book value of debt and the market value of equity; the book value of debt is the sum of short-term and long-term debt (item dlc plus item dltt). Asset growth is the annual relative change in total assets (item at). Sales growth is the annual relative change in net sales (item sale). Operating profitability is measured by the ratio of total revenue (item revt) or sales (item sale) minus cost of goods sold (item cogs) minus selling, general and administrative expense (item xsga) minus interest and related expense (item xint) to the book value of equity. Return on assets is the ratio of income before extraordinary items (item ib) to lagged total assets.

B.5 Additional tables and figures

[41] To minimize the instances of missing value, I impute missing items using other related items based on accounting identities whenever possible. For example, if item seq is missing, I use item ceq plus item pstk, or item at minus item lt minus item mib instead.

(a) *StG* (b) Market capitalization (c) Book-to-market (d) Market leverage

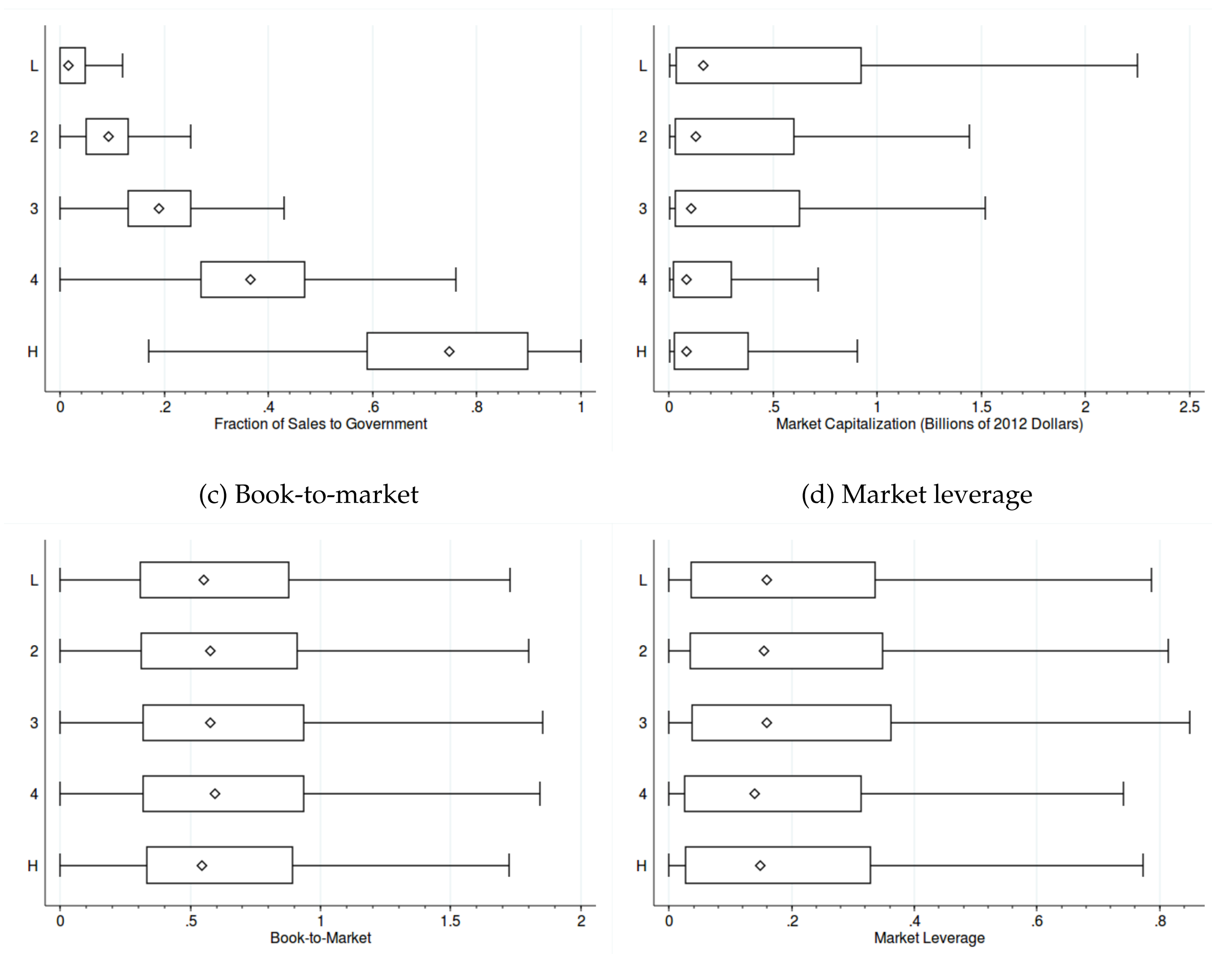
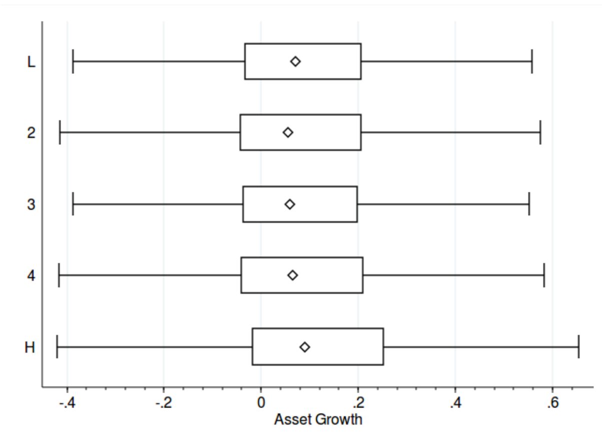
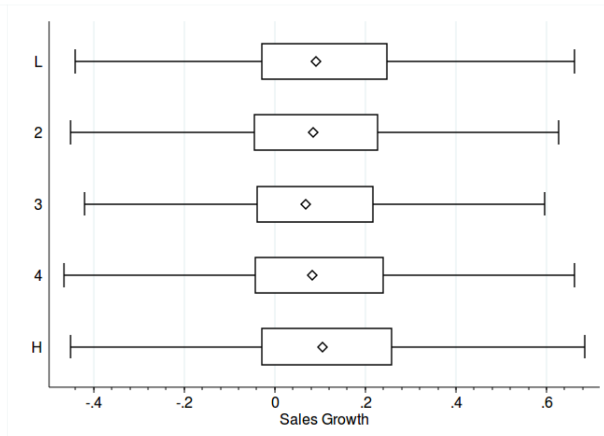


Figure B.1: **Firm characteristics across government dependency portfolios.** This figure compares via box plots the distributional properties of a selection of firm characteristics across portfolios formed on government dependency. In each panel, diamonds mark the medians of the corresponding characteristic, boxes span from the first to third quartiles, whiskers extend to the upper and lower adjacent values as defined by Tukey (1977). Detailed sample construction and variable calculations are in Appendix B.

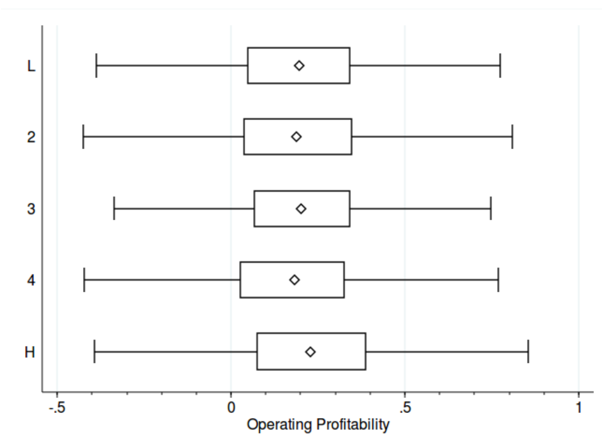
(e) Asset growth



(f) Sales growth



(g) Operating profitability



(h) Return on assets

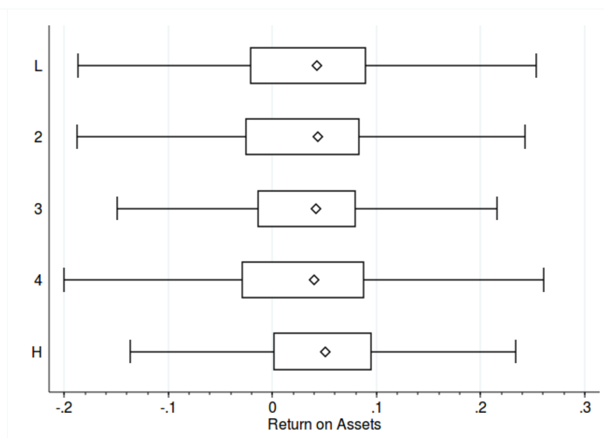
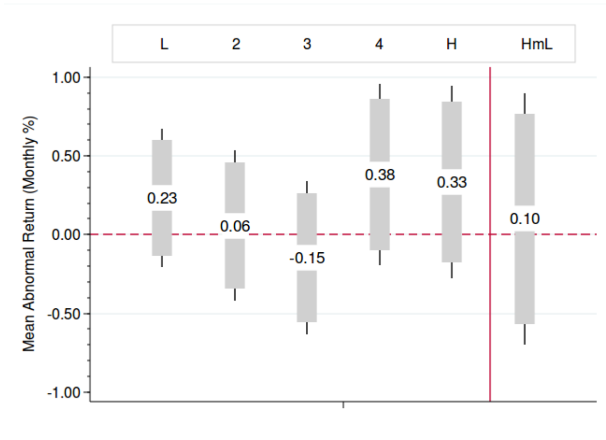
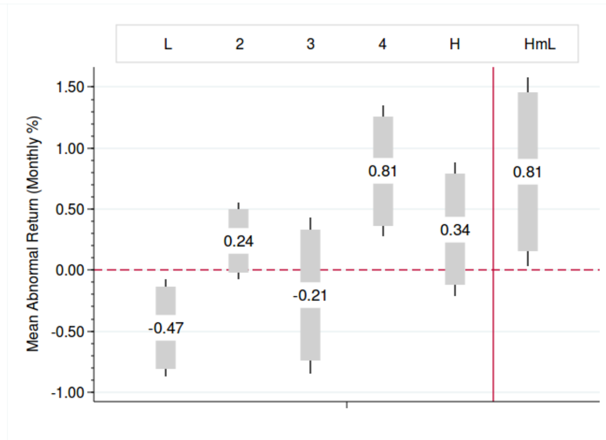


Figure B.1: *(Continued)*

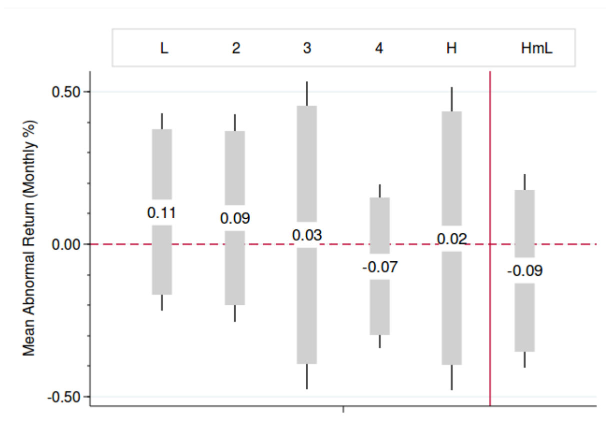
(a) Seven-factor alphas (v.w., 1981-1999)



(b) Seven-factor alphas (v.w., 2000-2018)



(c) Seven-factor alphas (e.w., 1981-1999)



(d) Seven-factor alphas (e.w., 2000-2018)

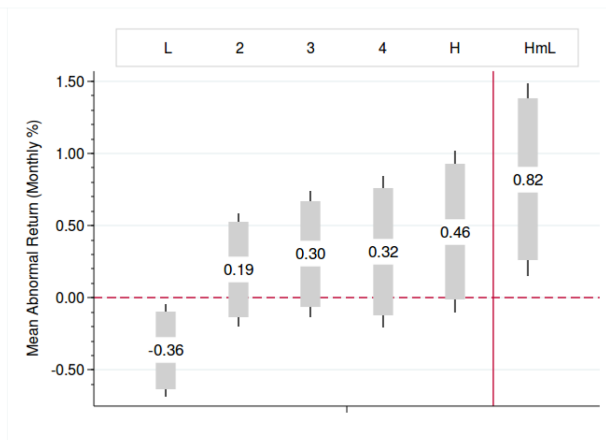


Figure B.2: **Government dependency portfolios: controlling for more risk factors.** This figure displays the alphas estimated by regressing the excess returns on government dependency portfolios as well as the return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile on seven classic risk factors including the market, size, and value factors from Fama and French (1993); the momentum factor from Carhart (1997); the liquidity factor from Pastor and Stambaugh (2003); and the profitability and investment factors from Fama and French (2015). Also displayed are 90% and 95% confidence intervals (indicated by the grey bar and the whiskers, respectively) computed with heteroskedasticity- and autocorrelation-consistent (HAC) standard errors following the routine of Newey and West (1987, 1994). Returns are monthly. Portfolios are value-weighted in panel (a) and (b), and equal-weighted in panel (c) and (d). The sample period is 1981 to 1999 in panel (a) and (c), and 2000 to 2018 in panel (b) and (d). The first portfolio formation was at the end of June in 1981, and it was based on government dependency ($\overline{StG}_{-2,0}$) computed for 1980; the same procedure are repeated every year thereafter until 2018.

Table B.2: **Government dependency portfolios: controlling for more risk factors; value-weighted portfolios.** This table presents the estimation results of regressing the excess returns on government dependency portfolios as well as the return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile on seven classic risk factors including the market, size, and value factors (MKT, SMB, HML) from Fama and French (1993); the momentum factor (MOM) from Carhart (1997); the liquidity factor (LIQ) from Pastor and Stambaugh (2003); and the profitability and investment factors (RMW, CMA) from Fama and French (2015). Panel (a) reports the alphas estimated separately for two subperiods, 1981-1999 and 2000-2018. Panel (b) reports the betas estimated for the full sample period, 1981-2018. Portfolios are value-weighted. Other specifics are the same as in Table 6.

	Govt. dependency portfolios					High *minus* Low
	1 (low)	2	3	4	5 (high)	
			(a) Alphas			
α (1981-1999)	0.23	0.06	-0.15	0.38	0.33	0.10
	[1.05]	[0.24]	[-0.60]	[1.31]	[1.08]	[0.25]
α (2000-2018)	-0.47	0.24	-0.21	0.81	0.34	0.81
	[-2.33]	[1.50]	[-0.64]	[2.99]	[1.21]	[2.06]
			(b) Betas			
β_{MKT}	1.14	1.13	1.25	0.88	0.94	-0.20
	[22.82]	[21.92]	[20.53]	[12.89]	[19.16]	[-3.53]
β_{SMB}	0.21	0.18	0.22	0.27	0.33	0.12
	[2.30]	[2.20]	[3.13]	[2.49]	[3.13]	[1.00]
β_{HML}	0.09	-0.09	-0.05	-0.11	0.14	0.06
	[1.13]	[-0.78]	[-0.53]	[-0.66]	[1.16]	[0.39]
β_{MOM}	0.00	-0.19	-0.16	-0.03	0.04	0.04
	[-0.06]	[-2.99]	[-3.06]	[-0.59]	[0.56]	[0.46]
β_{LIQ}	0.19	0.01	0.13	0.00	-0.10	-0.28
	[2.13]	[0.19]	[1.46]	[-0.00]	[-1.22]	[-2.45]
β_{RMW}	-0.26	0.04	0.25	-0.53	0.44	0.70
	[-2.48]	[0.29]	[2.30]	[-2.22]	[3.78]	[6.17]
β_{CMA}	-0.24	0.20	0.44	-0.12	0.09	0.33
	[-2.07]	[1.04]	[2.60]	[-0.68]	[0.58]	[1.71]
Adj. R^2	0.67	0.68	0.67	0.53	0.49	0.18

Table B.3: **Government dependency portfolios: controlling for more risk factors; equal-weighted portfolios.** This table presents the estimation results of regressing the excess returns on government dependency portfolios as well as the return on a zero-investment portfolio that is long stocks in the highest-dependency quintile and short stocks in the lowest-dependency quintile on seven classic risk factors including the market, size, and value factors (MKT, SMB, HML) from Fama and French (1993); the momentum factor (MOM) from Carhart (1997); the liquidity factor (LIQ) from Pastor and Stambaugh (2003); and the profitability and investment factors (RMW, CMA) from Fama and French (2015). Panel (a) reports the alphas estimated separately for two subperiods, 1981-1999 and 2000-2018. Panel (b) reports the betas estimated for the full sample period, 1981-2018. Portfolios are equal-weighted. Other specifics are the same as in Table 6.

	Govt. dependency portfolios					High *minus* Low
	1 (low)	2	3	4	5 (high)	
			(a) Alphas			
α (1981-1999)	0.11	0.09	0.03	-0.07	0.02	-0.09
	[0.64]	[0.50]	[0.12]	[-0.53]	[0.08]	[-0.54]
α (2000-2018)	-0.36	0.19	0.30	0.32	0.46	0.82
	[-2.22]	[0.98]	[1.37]	[1.19]	[1.60]	[2.41]
			(b) Betas			
β_{MKT}	1.01	0.91	0.95	0.86	0.83	-0.18
	[28.71]	[23.58]	[28.91]	[19.62]	[15.89]	[-3.68]
β_{SMB}	0.80	0.84	0.79	1.00	0.82	0.03
	[12.11]	[9.59]	[10.35]	[17.19]	[10.52]	[0.47]
β_{HML}	0.10	-0.01	-0.07	-0.05	-0.07	-0.17
	[1.24]	[-0.15]	[-1.00]	[-0.57]	[-0.51]	[-1.75]
β_{MOM}	-0.13	-0.25	-0.17	-0.15	-0.11	0.03
	[-2.78]	[-5.97]	[-2.86]	[-5.18]	[-1.86]	[0.72]
β_{LIQ}	0.09	-0.03	-0.02	0.05	-0.08	-0.17
	[1.54]	[-0.75]	[-0.48]	[0.83]	[-1.71]	[-3.08]
β_{RMW}	-0.07	-0.13	-0.08	-0.06	0.20	0.27
	[-0.97]	[-2.03]	[-1.38]	[-0.85]	[2.31]	[3.32]
β_{CMA}	-0.14	-0.02	0.13	0.02	0.01	0.15
	[-1.28]	[-0.20]	[1.60]	[0.20]	[0.05]	[1.01]
Adj. R^2	0.80	0.79	0.80	0.78	0.67	0.11

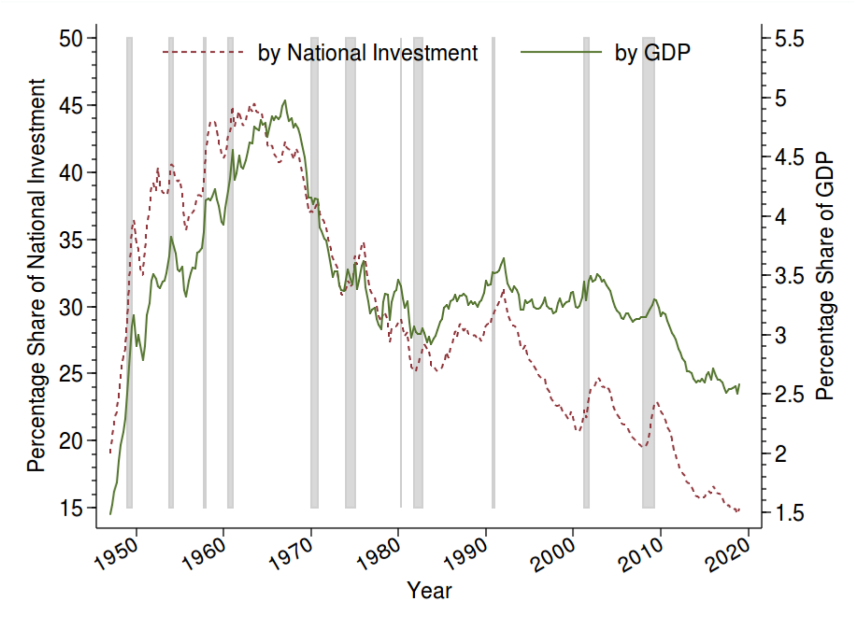


Figure B.3: **Public sector investment share: relative to GDP.** The solid line represents an alternative definition of the public sector investment share, which is the ratio of public sector investment to GDP; it is compared with the original definition denoted by the dashed line. Shaded areas indicate U.S. recessions identified by NBER.

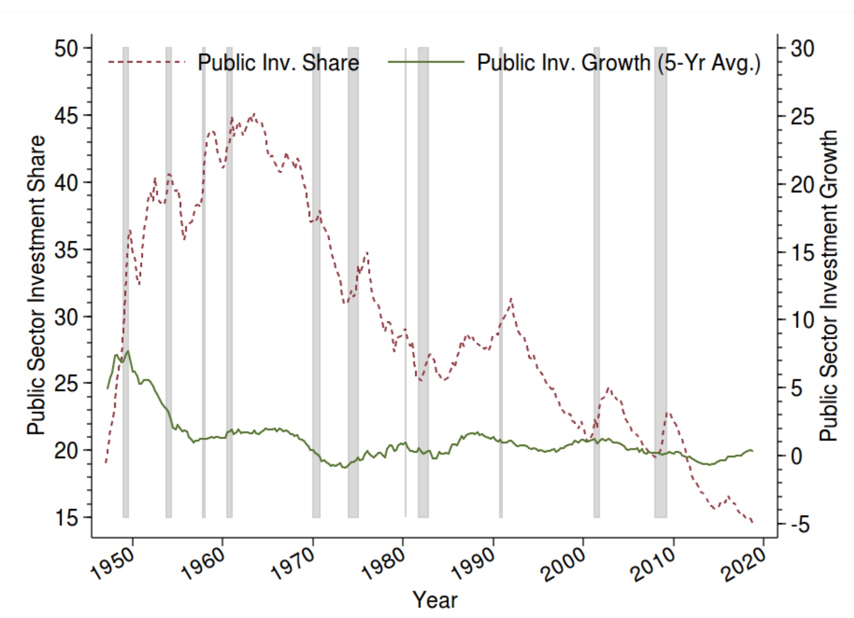


Figure B.4: **Public sector investment growth.** The solid line represents the average growth rate of public sector investment over the past 5 years. it is compared with the public sector investment share denoted by the dashed line. Shaded areas indicate U.S. recessions identified by NBER.

Table B.4: **Mean excess returns and β_{Pub} for 25 Size-(Inv/OP/Mom) equity portfolios.** This table reports the test assets' mean excess returns ($\overline{r_i^e}$) and estimated β_{Pub}. The latter are obtained by running a time-series regression specified as $r_{i,t}^e = a_i + \boldsymbol{f}_t' \boldsymbol{\beta}_i + \xi_{i,t}$ for each asset i, where $r_{i,t}^e$ is the asset's excess return, $\boldsymbol{f}_t$ represents a vector of risk factors including $PubFac$, $UncFac$ and the market excess return, and $\boldsymbol{\beta}_i$ denotes a vector of beta estimates. The test assets include 25 size and investment (Inv) or profitability (OP) or momentum (Mom) sorted equity portfolios. The sample is quarterly and spans the period 1960Q4 to 2018Q4.

		Size									
		Small				Big	Small				Big
				$\overline{r_i^e}$					β_{Pub}		
	Low	2.75	2.58	2.53	2.26	2.06	0.72	0.76	0.20	0.05	0.61
		2.79	2.63	2.65	2.16	1.61	0.76	0.22	0.38	0.56	0.05
Inv		2.86	2.66	2.35	2.19	1.48	0.61	0.55	0.39	0.32	-0.32
		2.51	2.62	2.36	2.22	1.48	0.67	0.85	0.31	0.09	-0.48
	High	1.02	1.42	1.48	1.65	1.43	1.22	0.75	0.43	0.17	-0.03
				$\overline{r_i^e}$					β_{Pub}		
	Low	1.55	1.74	1.61	1.63	1.05	0.83	0.44	0.10	0.26	-0.58
		2.74	2.33	2.19	2.00	1.06	0.44	0.47	-0.18	0.09	0.23
OP		2.53	2.38	2.15	1.95	1.52	0.78	0.42	0.56	0.24	-0.12
		2.81	2.29	2.16	2.13	1.43	0.91	1.07	0.55	0.40	-0.17
	High	2.37	2.75	2.64	2.36	1.78	1.35	0.81	0.77	0.37	-0.18
				$\overline{r_i^e}$					β_{Pub}		
	Low	0.08	0.48	0.73	0.57	0.49	1.56	1.07	1.02	0.85	0.13
		1.96	2.00	1.78	1.74	1.39	0.77	0.69	0.51	0.72	0.35
Mom		2.80	2.44	2.11	2.00	1.34	0.39	0.58	0.55	0.46	0.02
		3.19	2.95	2.23	2.38	1.69	0.42	0.44	0.03	0.03	-0.07
	High	3.92	3.57	3.41	3.06	2.40	0.79	0.14	0.01	-0.17	-0.03